

Evaluating Corporate Bonds with Complex Debt Structure

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Outline

- Motivation
- Main Contributions
- Assumptions and Numerical Implementation
 - **market and model assumptions**
 - **tree and forest**
- Numerical Results and Empirical Implication
 - **4 numerical simulations**
- Conclusions

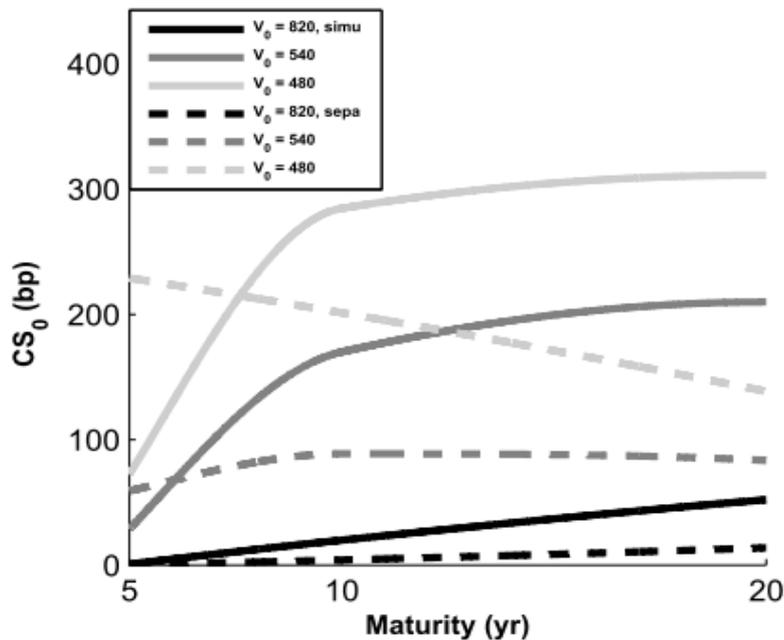
Motivation

- **A corporate bond is a popular financing or investment tool.**
- **Among valuation methodologies,**
the structural model of credit risk is a popular approach
that associates bond prices with firm-specific capital structures
- **Regarding the capital structure,**
 - **empirical observations: equity + heterogeneous bonds**
 - **theoretical researches: equity + uniform debt**
- **Regarding bond evaluation with structural models,**
 - **conventional structural models separately evaluate each bond of the same firm to proxy real world observations**

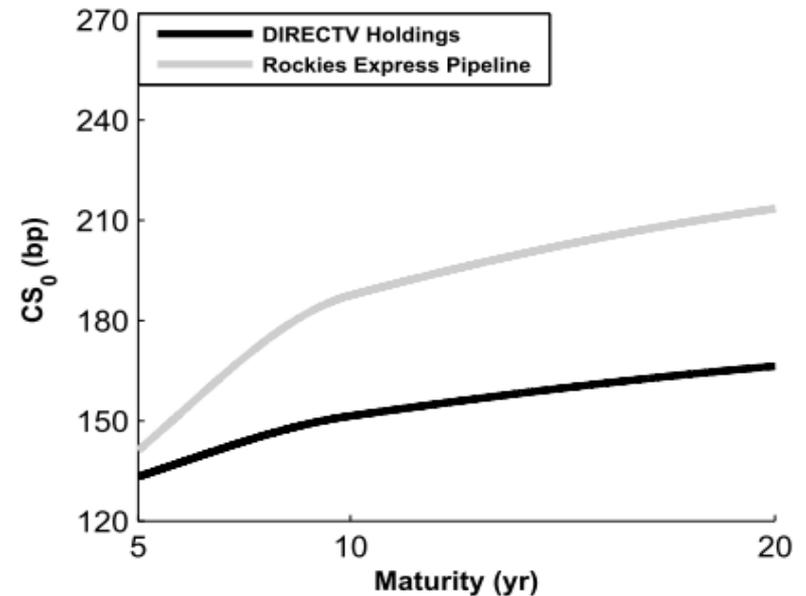
- To move structural models in the empirically observed direction, we address that

- rather than being evaluated separately, corporate bonds issued by the same firm must be priced considering the presence of other simultaneously outstanding bonds of that firm

solid curve: bonds are simultaneously evaluated
dash curve: bonds are separately evaluated



real world observations



Main Contributions

- **A more intuitive valuation framework can be implemented!**
 - That is, corporate bonds of the same firm are priced considering the presence of other simultaneously existing bonds with different properties, such as different maturities, seniorities,..etc.

- **A structural model that characterizes the multidimensionality of corporate debt structure is developed!**
 - Four observable dimensions: (1) leverage ratio (2) maturity structure (3) priority structure (4) covenant structure

(Helwege and Turner, 1999; Kisgen, 2006; Billett et al. 2007; Mauer et al., 2012)

- **To implement the structural model with debt structure, we propose a novel quantitative framework: a multi-layer forest.**
 - **To capture the contingent changes of the debt structure due to early redemption, we need more than one tree and make them work collaboratively.**
 - **It may be an alternative way to solve the unsolved problem in [Jones et al. \(1985\)](#)**

- **Regarding the feasibility of structural models, considering macroeconomic factors is important!**

(Collin-Dufresne et al., 2001)

**Given parameters, we show that
considering corporate debt structure is also important!**

- **This may greatly reconcile structural models with empirical observations.
(respond to Davydenko(2012))**
- **Our valuation framework will provide theoretical insights and concrete quantitative measurements on these empirical phenomena.**

- **Numerous phenomena are revisited through our valuation framework:**

[1] change of credit spreads **vs.** change of leverage ratio

$$- \Delta Lev_0 \uparrow \Rightarrow \Delta s \uparrow, E_0[\Delta Lev_t] \uparrow \Rightarrow \Delta s \uparrow$$

(\because claim dilution effect)

(Collin-Dufresne et al., 2001; Flannery et al., 2012)

[2] the impact of a junior bond issuance to replace a bank loan on the existing senior unsecured bond

- (1) $\Delta s_{senior} < 0$, the more pronounced the replacement size

(\because the relative priority of the senior bond improves)

(2) **ceteris paribus, the maturity of the junior bond makes small difference on average until the firm becomes relatively unhealthy**

(\because the existence of payment blockage covenant)

(Linn and Stock, 2005)

[3] call delay phenomena

- **(1) call delay exists due to market frictions**
- (2) much longer delay due to the wealth transfer effect**
- (3) wealth transfer effect would vanish due to the level of interest rate**

(Ingersoll 1977, Longstaff and Tuckman, 1994; King and Mauer, 2000)

[4] the effect of poison put covenants on bidders' cost of debt

- **the poison put covenants in the bonds of the target firm significantly increase the bond value of the target firm by increasing the bidders' costs of debt.**

(Cremers et al., 2007)

Assumptions and Numerical Implementation

- **Market / Model Assumptions**
- **Numerical Implementation: binomial/trinomial trees; forests**

Market Assumptions

(A.1) Prices are determined in the market place such that perfect substitutes are valued identically. **(No Dominated Assets!)**

(A.2) All market participants (including equity and bond holders) have equal access to information.

(No Information Asymmetry!)

(A.3) All market participants prefer more wealth to less. A firm manager acts to maximize the benefits of equity holders rather than overall firm value.

(Behaviors of Self-Interest Maximization)

(Agency Problem Exists!)

(A.4) There exists two types of market frictions:

(1) corporate income tax

(2) bankruptcy cost **(A Capital Market with Frictions!)**

(A.5) A firm files bankruptcy and is liquidated immediately once it defaults on its debt obligation. **(Immediate Liquidation!)**

The assets are distributed according to absolute priority rules
(No Violation of Absolute Priority Rules!)

Model Assumptions

- The Dynamics of the Firm Asset Value

- Under the structural model of credit risk, corporate bonds and the corresponding equity are contingent claims on the firm asset value.
- Denote the firm asset value at time t as V_t
- Under the risk-neutral valuation, the firm asset value follows the dynamics

$$dV_t = \underbrace{rV_t dt}_{\text{Diffusion Part}} + \underbrace{C_t^I - C_t^O}_{\text{Cash in-/out- flows}}, \quad (1)$$

where

(Merton (1974))

- [1] r is the long-term average risk-free rate;
 - [2] σ is the volatility of the firm asset value;
 - [3] dz is a standard Brownian motion;
 - [4] C_t^I captures the cash inflows realized at time t due to raises of equity or debt capitals;
 - [5] C_t^O captures the cash outflows realized at time t due to bond repayments or dividend payouts.
- A firm manager determines to file bankruptcy once $V_t \leq C_t^O$ for all $t > 0$

- Previous literatures impose additional assumptions on C_t^I and C_t^O

For example,

(a) **Geske (1977)** assumes $C_t^I - C_t^O = 0$ for all $t > 0$, implying all repayments are financed by issuing new equities.

(b) **Leland and Toft (1996)** assumes $C_t^I - C_t^O = -\delta V_t dt$ for all $t > 0$.

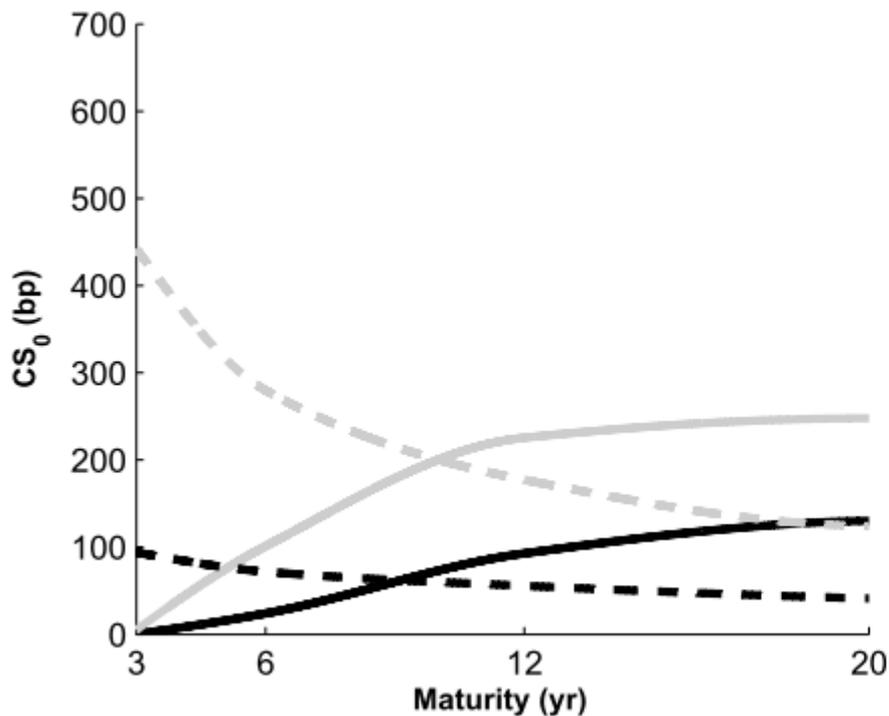
- The firm will rollover all its bonds to keep the number of its outstanding bonds, sums of bond principals and annual coupon payments unchanged.
- The equity holders will absorb all deficiencies in required payments, including the rollover losses, to prevent bankruptcy.

Note that the firm is liquidated once its equity value is less than the required payments.

- However, these assumptions will lead to biased results, such as hump-shaped or downward-sloping credit spread curves. **(Leland and Toft, 1996; Lando, 2004; Eom et al, 2004)**

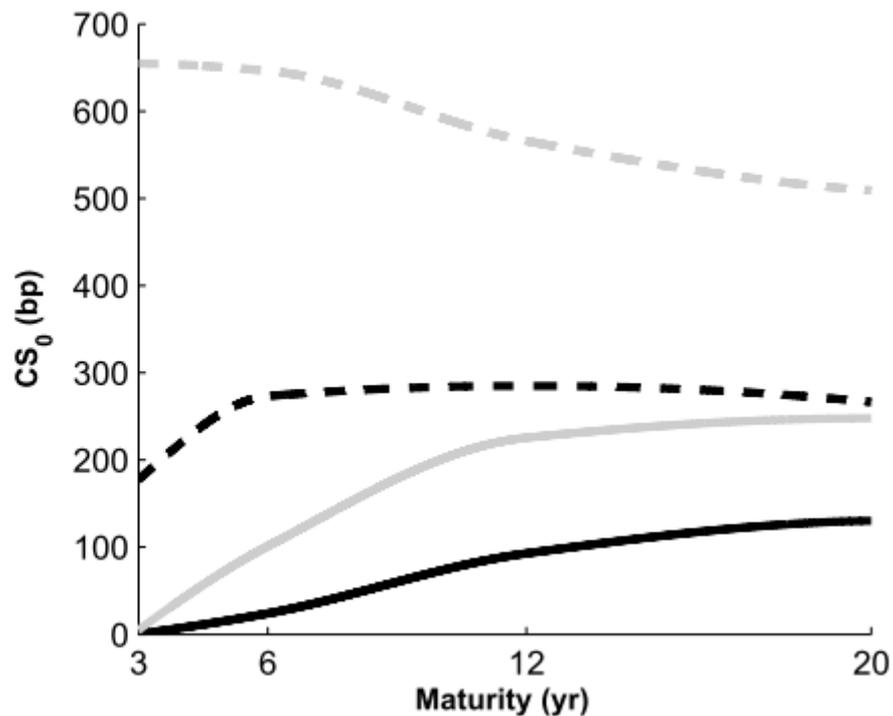
For example, with the same parameters,

solid curve: our valuation framework
dash curve : with Geske's assumption



(a)

solid curve: our valuation framework
dash curve : with Leland's assumption



(b)

Numerical Implementation

- binomial/trinomial trees

- CRR Trees for the lognormal diffusion process:

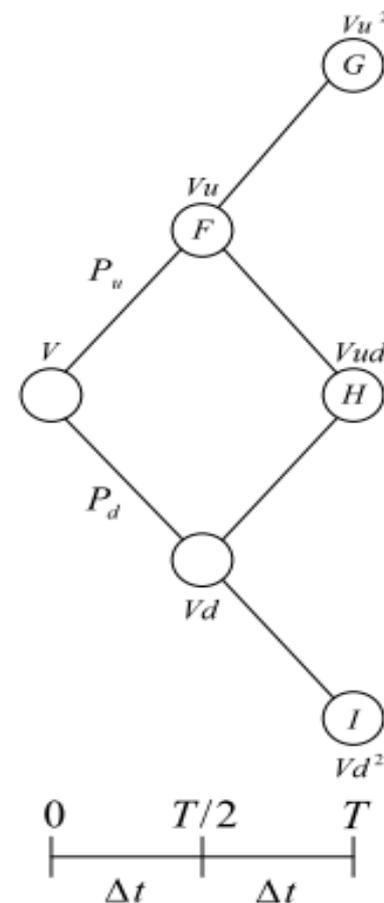
$$dV_t = rV_t dt + \sigma V_t dz$$

[1] Size of one time step: $\Delta t = T/n$

[2] 4 parameters: u, d, P_u, P_d :

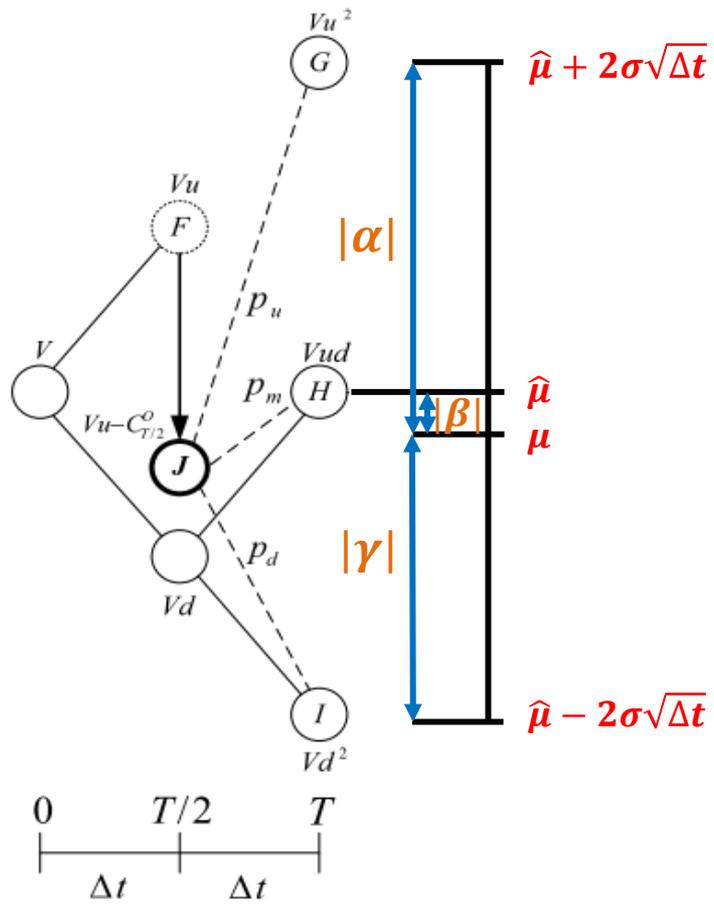
- $u = e^{\sigma\sqrt{\Delta t}}, d = 1/u;$

- $P_u = \frac{e^{r\Delta t} - d}{u - d}, P_d = 1 - P_u.$





- The trinomial structure is used to deal with jumps in a firm's asset value:



- With feasible branching probabilities p_u, p_m, p_d that satisfying

$$\begin{cases} p_u \alpha + p_m \beta + p_d \gamma = 0 \\ p_u (\alpha)^2 + p_m (\beta)^2 + p_d (\gamma)^2 = \sigma^2 \Delta t \\ p_u + p_m + p_d = 1 \end{cases}$$

where

$$\begin{aligned} \alpha &\equiv \beta + 2\sigma\sqrt{\Delta t} \\ \beta &\equiv \hat{\mu} - \mu \\ \gamma &\equiv \beta - 2\sigma\sqrt{\Delta t} \end{aligned}$$

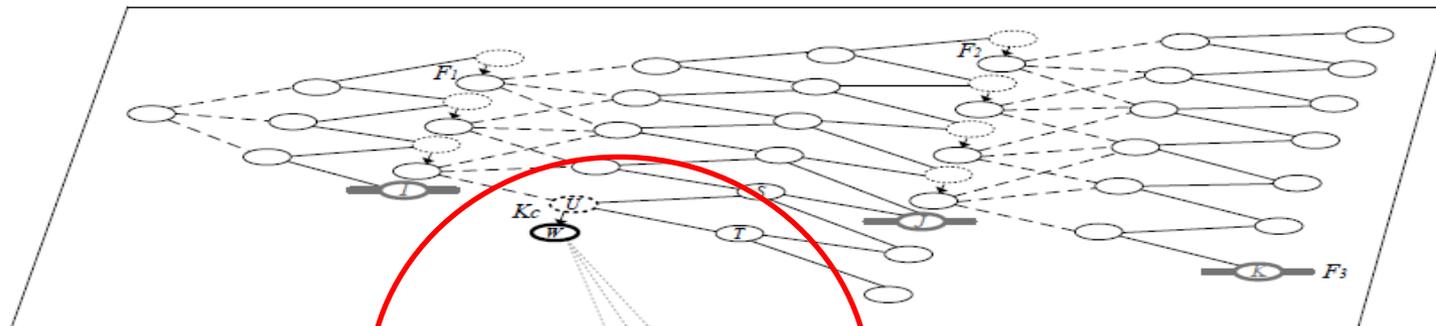
Dai and Lyuu (2010)

Numerical Implementation

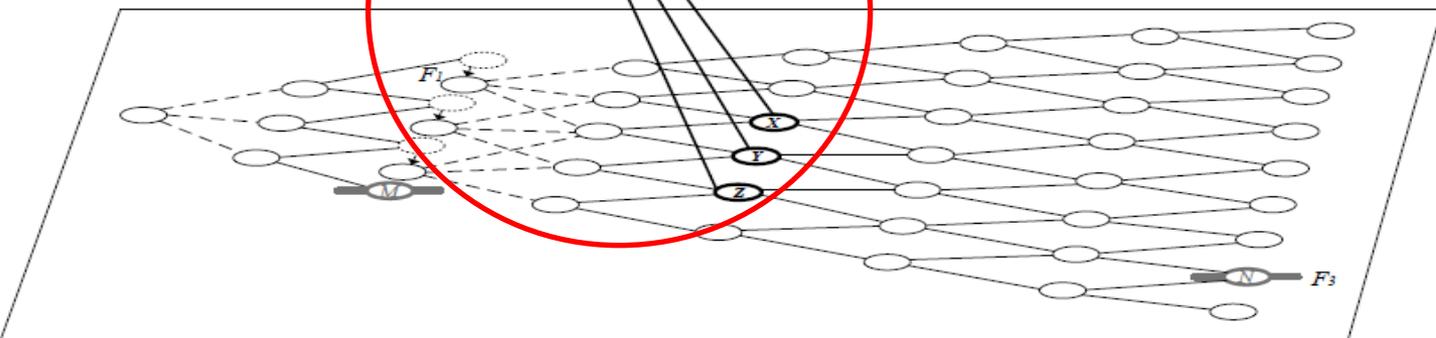
- the forest

- To capture the contingent changes of the debt structure due to early redemption, we need a forest.
- For example, a firm issues 2 straight bonds and 1 callable bond,

0 $T/7$ $2T/7$ $3T/7$ $4T/7$ $5T/7$ $6T/7$ T



2 straights and 1 callable
(before redemption)



2 straights
(after redemption)

A two-level forest

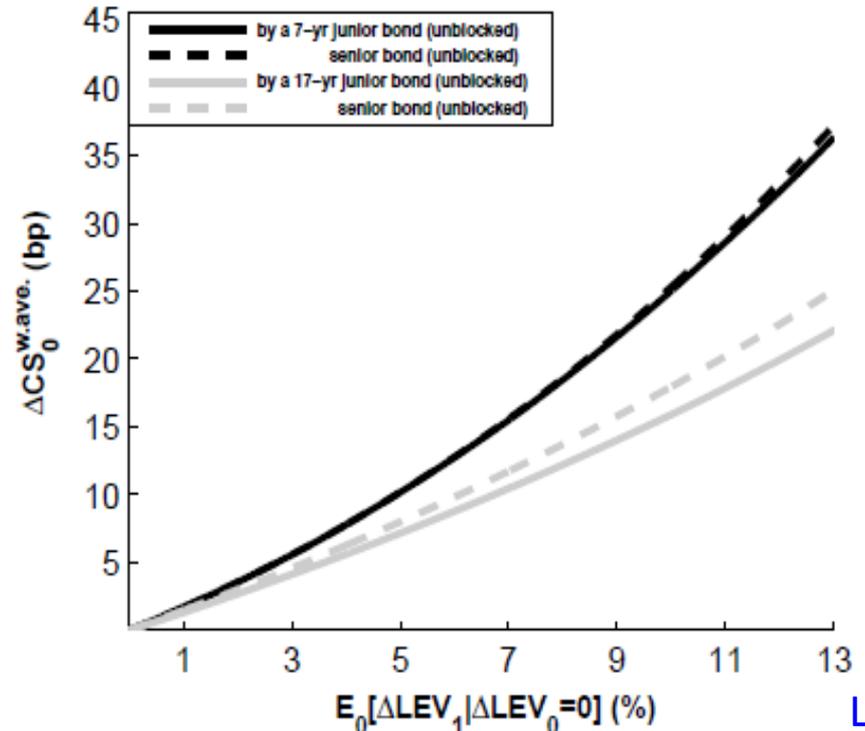
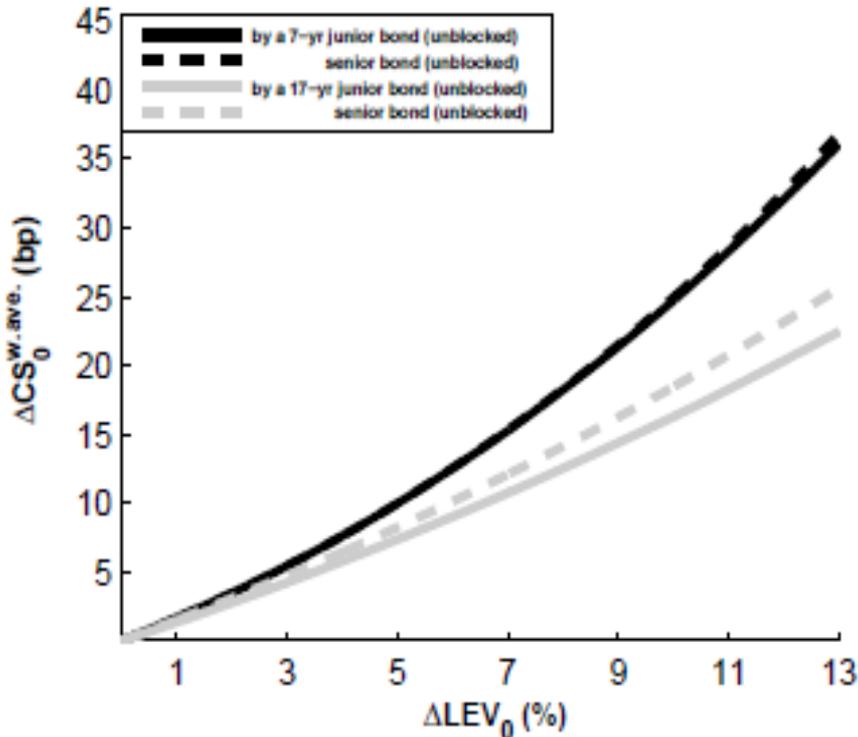
Numerical Results and Empirical Implication

- [1] change of credit spreads vs. change of leverage ratio**
- [2] the effect of junior bond issuances to replace bank loans on senior unsecured bonds**
- [3] call delay phenomena**
- [4] the effect of poison put covenants on bidders' cost of debt**

change of credit spreads vs. change of leverage ratio

- **Debt structure:** (1) 5-yr bank loan (BD), 10-yr senior debenture (SD), 20-yr junior debenture (JD), with equal face value
(2) $BD \succ SD \succ JD$

Scenario: The firm announces a new bond issuance, a new SD or JD, now or in the future (1 yr)

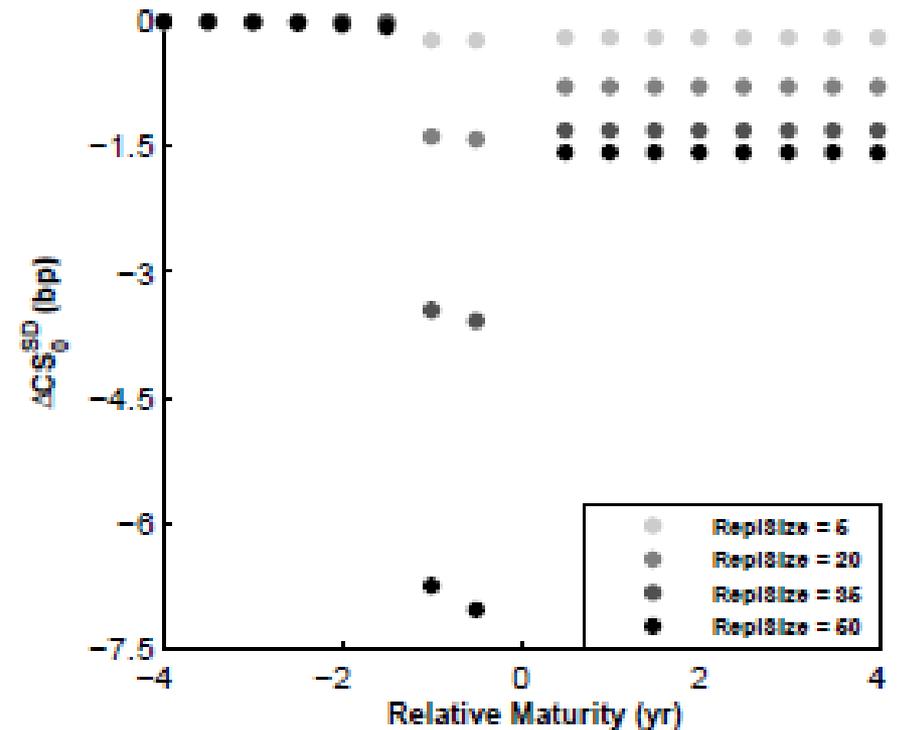
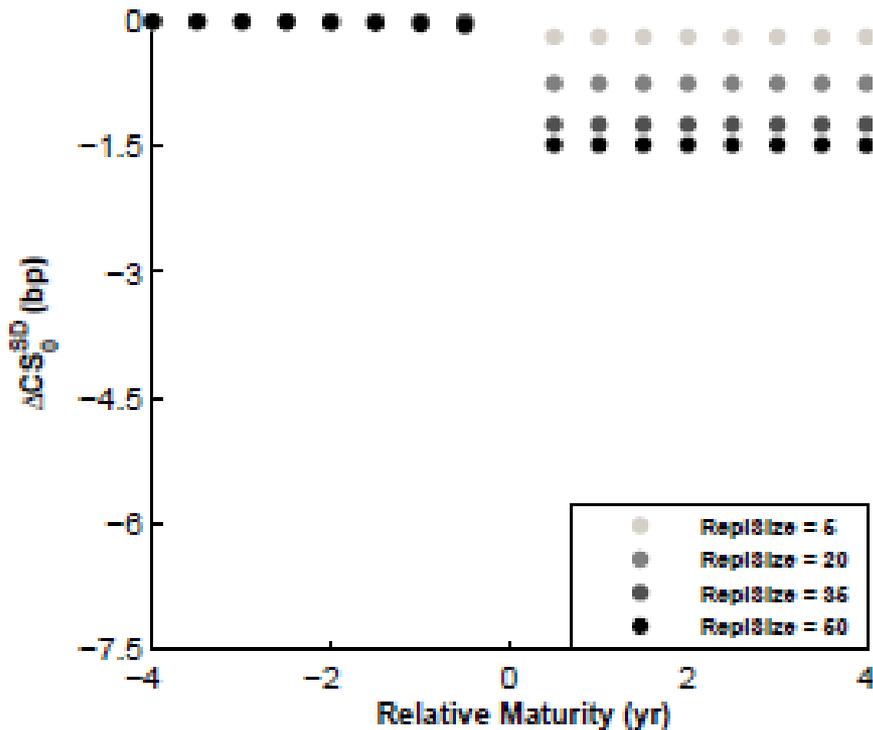


the effect of junior bond issuances to replace a bank loan on a senior unsecured bond

- **Debt structure:** (1) BD, 10-yr SD, 20-yr JD, given SD's face value
(2) $BD > SD > JD$

Scenario: The firm now issues an otherwise identical new JD to replace the existing BD

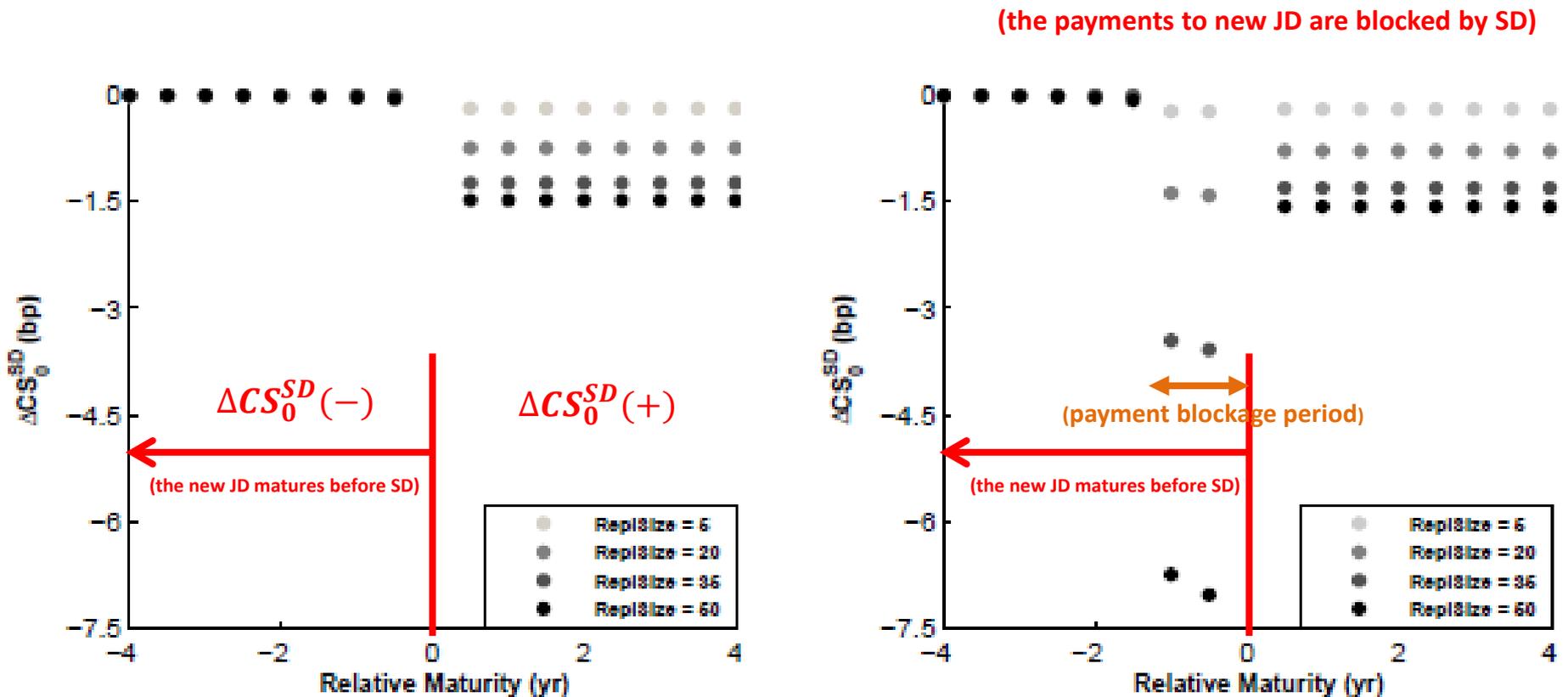
(the payments to new JD are blocked by SD)

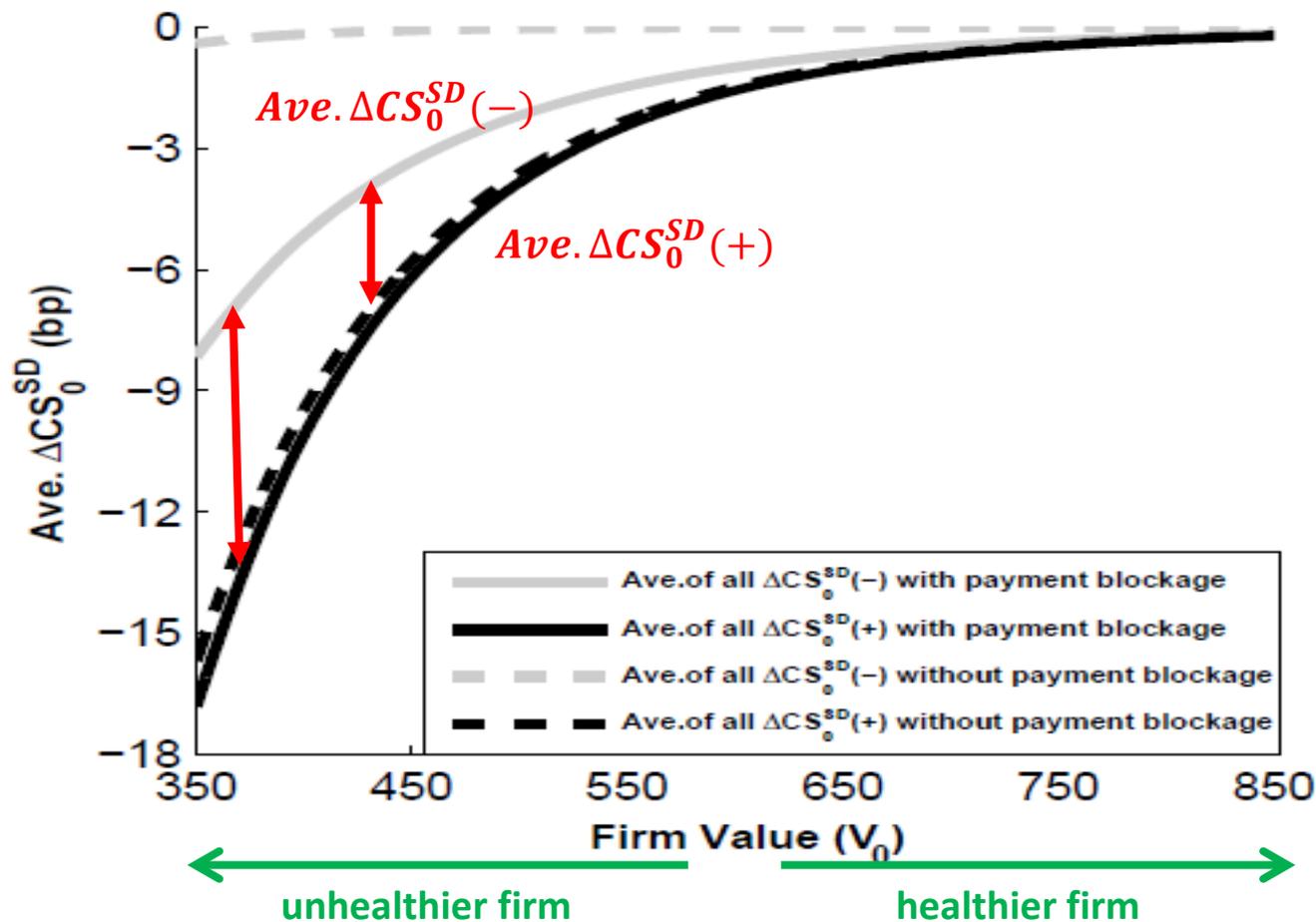


the effect of junior bond issuances to replace a bank loan on a senior unsecured bond

- **Debt structure:** (1) BD, 10-yr SD, 20-yr JD, given SD's face value
 (2) $BD > SD > JD$

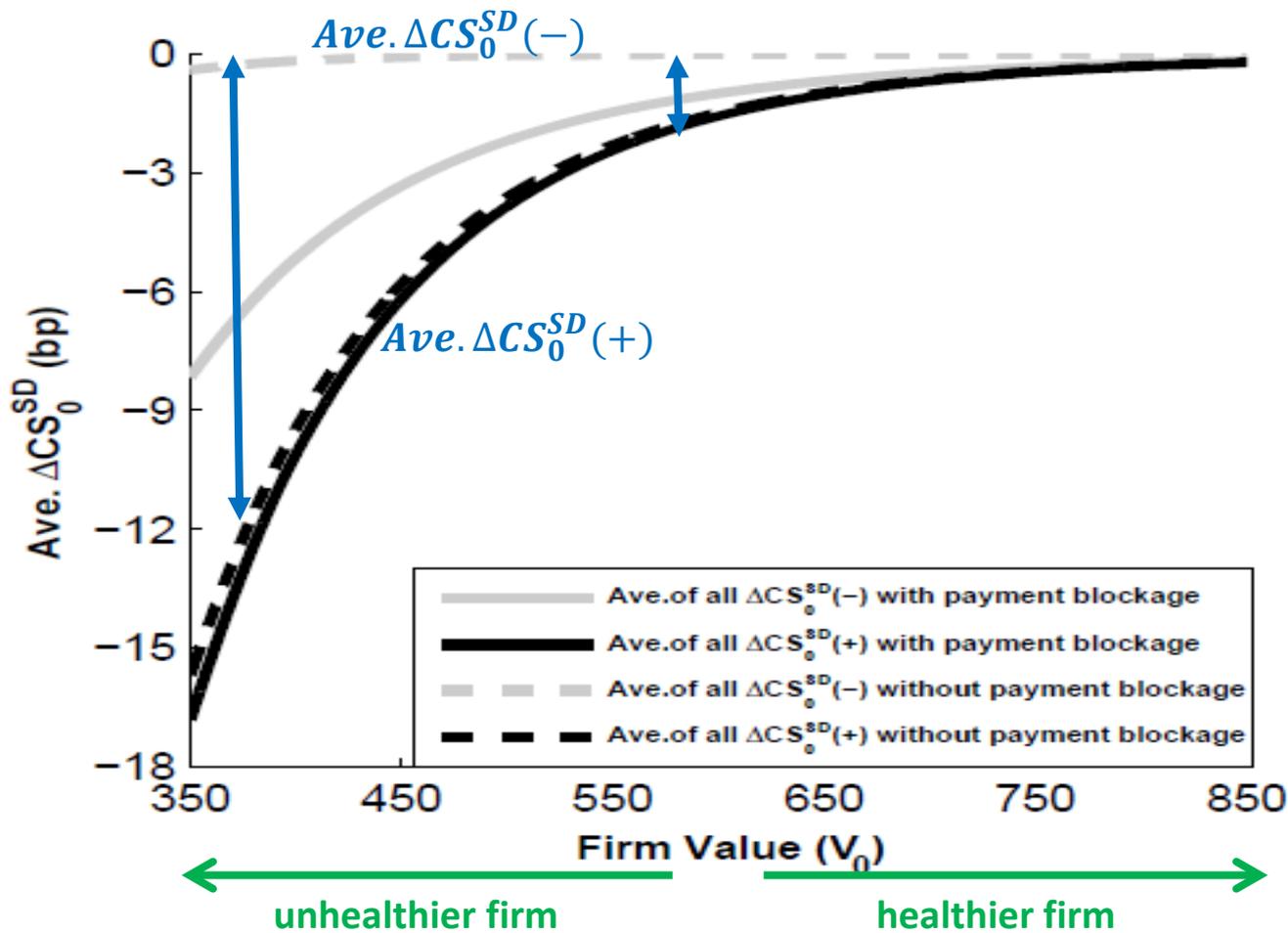
Scenario: The firm now issues an otherwise identical new JD to replace the existing BD





Solid Curves
(with payment blockage)

With the aforementioned effect given a replacement size, the maturity of the junior bond issuance makes small difference when the firm is healthy, but makes pronounced difference when the firm is relatively unhealthy

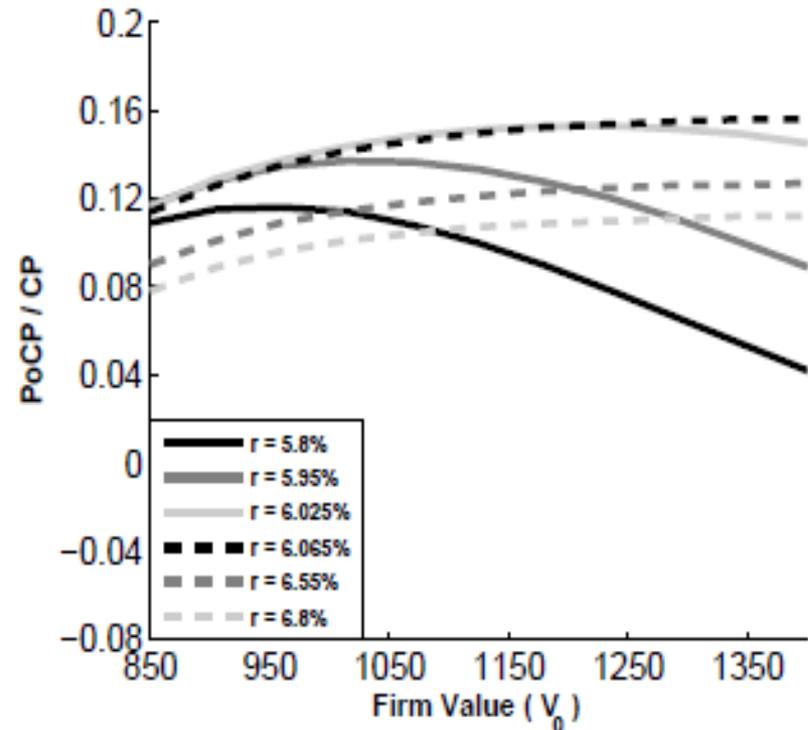
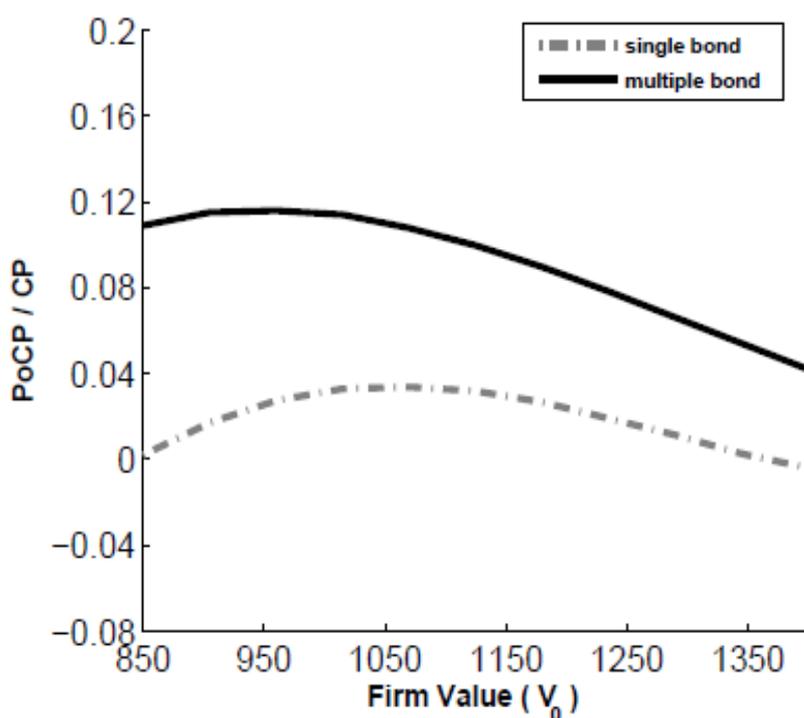


Dashed Curves
(no payment blockage)

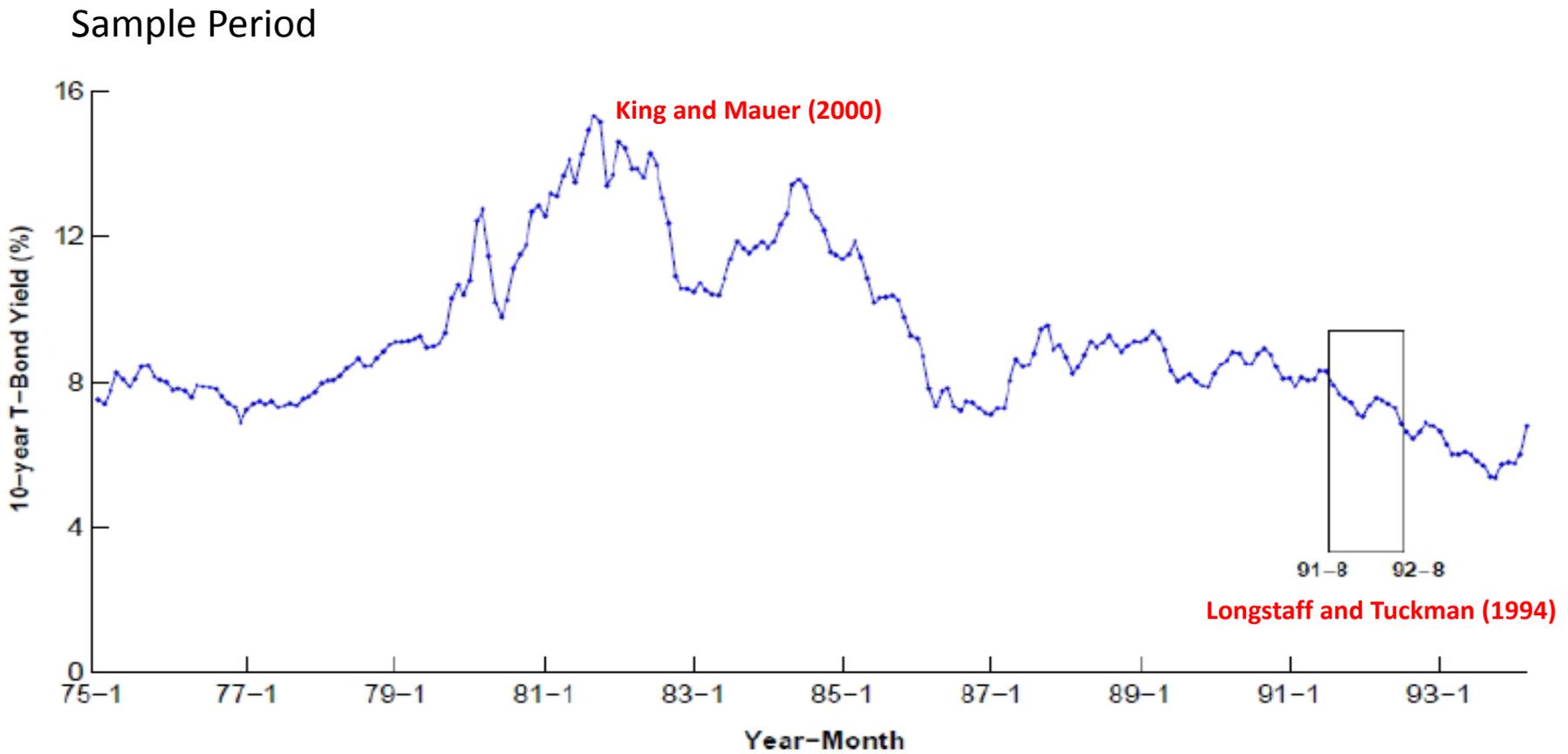
The difference is much more significant despite the credit quality of the firm

Call Delay Phenomena

- **Debt structure:** (1) a 7-yr callable bond
(2) 5 otherwise identical 3-yr, 5-yr, 7yr, 9-yr and 12-yr bonds, the 7-yr bond is callable.
- **Scenario:** compare the premium over effective call price (PoCP) of the callable bond in (1) with that in (2), other things being equal.



- **Longstaff and Tuckman (1994)** predict hump-shaped curve
King and Mauer (2000) observe upward-sloping curve

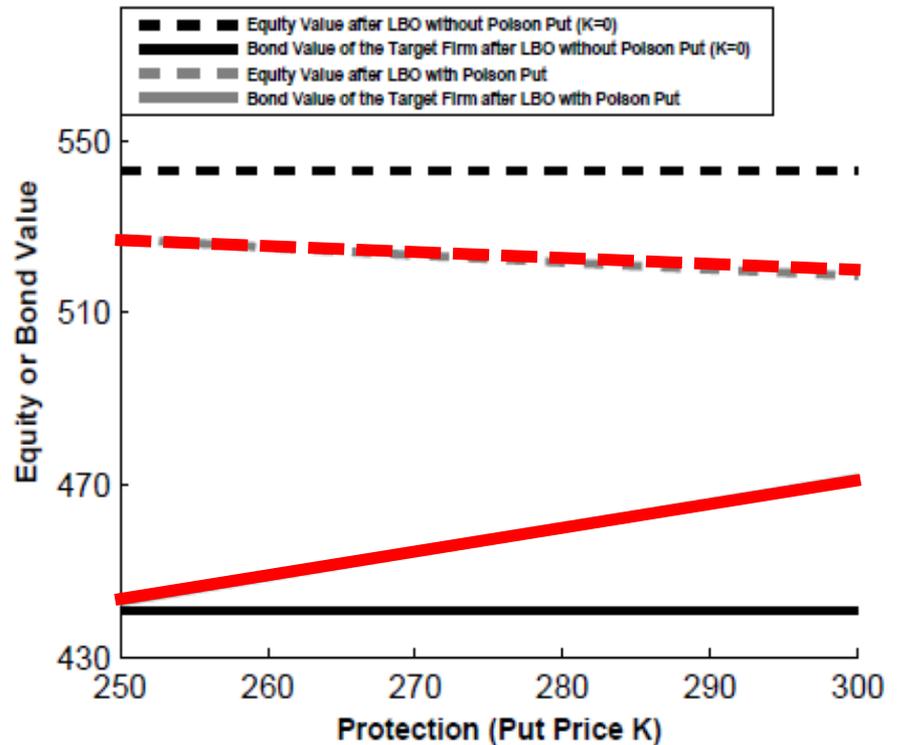
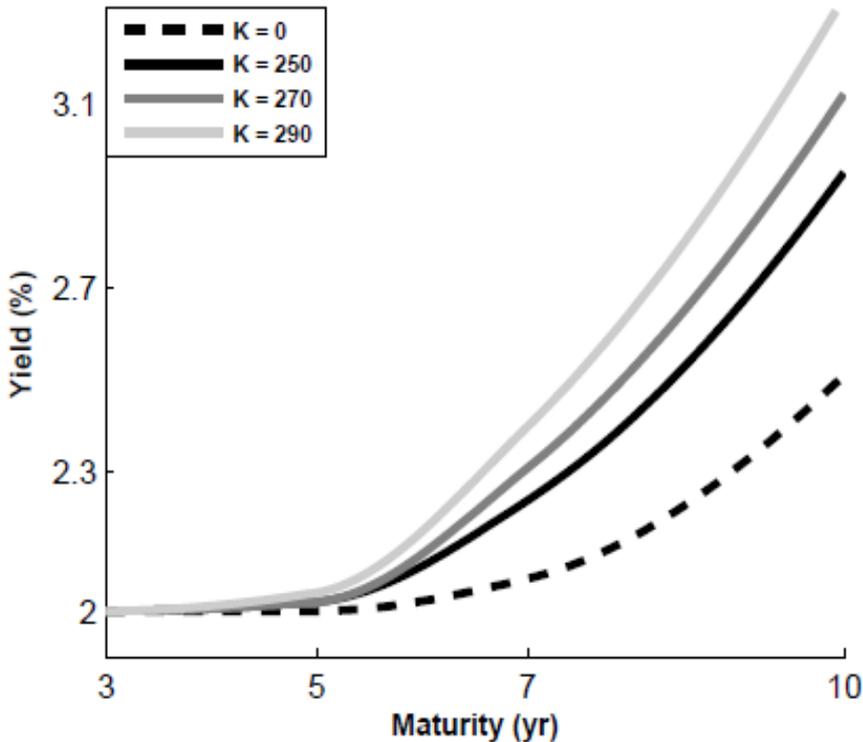


the effect of poison put covenants on bidders' cost of debt

- **Scenario:** A bidder accomplishes a LBO by issuing 4 otherwise identical bonds: 3-yr, 5-yr, 7-yr and 10-yr.

The original bidder's bond is a 2-yr bond with 10% coupon

The original target firm's bond is a 20-yr bond with 10% coupon and lowest priority



Conclusions

- A structural model of credit risk with debt structure simultaneously considering four observable dimensions is developed:
 - **[1]** leverage ratio **[2]** maturity structure
[3] priority structure **[4]** covenant structure
 - the forest can capture the contingent changes of debt structure, such as the early redemptions by bonds with call provisions or poison put covenants

- Compared with existing structural models, our valuation framework can produce the results that are more consistent with the observations documented already in empirical studies.

- including 5 observations:

[1] upward sloping credit spread curves

[2] change of credit spreads vs. change of leverage ratio

[3] the effect of junior bond issuances to replace bank loans on senior unsecured bonds with or without payment blockage covenants

[4] call delay phenomena considering the wealth transfer effect

[5] the effect of poison put covenants on bidders' cost of debt

- Actually, our valuation framework can produce more results that are consistent with observations:

[6] the impacts of rollover risk on a firm's existing long-term bonds

(Gopalan et al., 2014, Nagler, 2014)

[7] the debt structure with multiple callable bonds:

(1) multiple calls exist

(2) ceteris paribus, the long-term bond is called first

(3) the pattern that characterizes wealth transfer effect

may also vanish due to

(a) the existence of call protection

(b) the existence of long and short callable bonds

(King and Mauer, 2000; Sarkar, 2003)

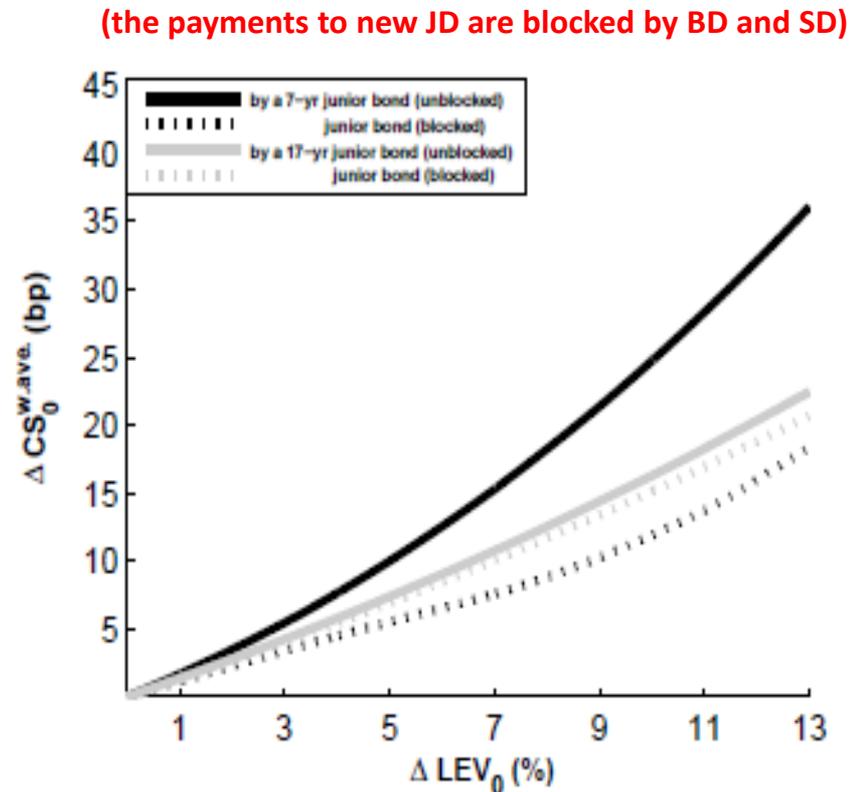
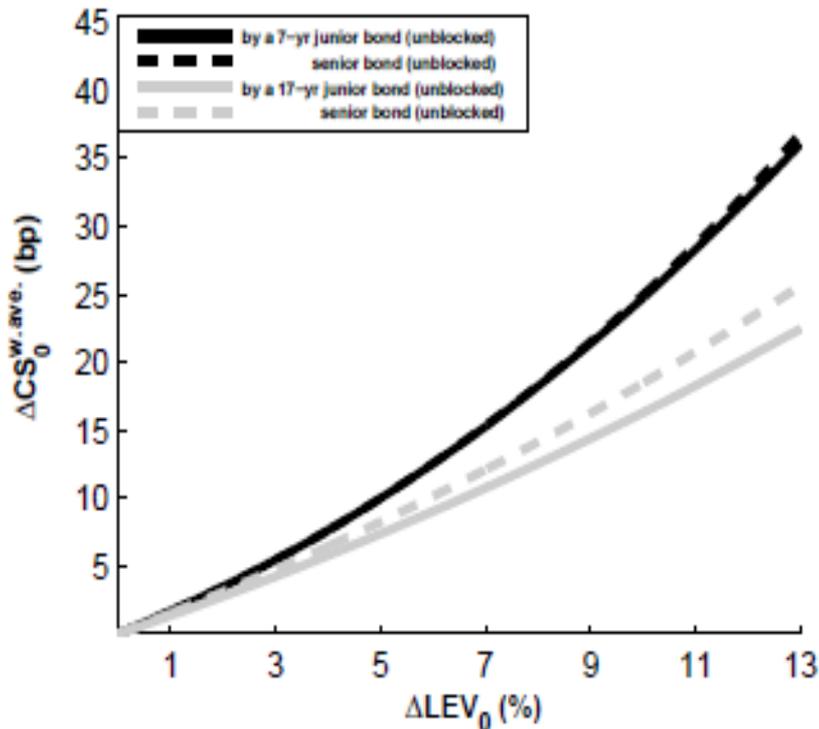
Appendix 1

**change of credit spreads vs. change of leverage ratio -
with or without payment blockage covenants in BD & SD**

change of credit spreads vs. change of leverage ratio

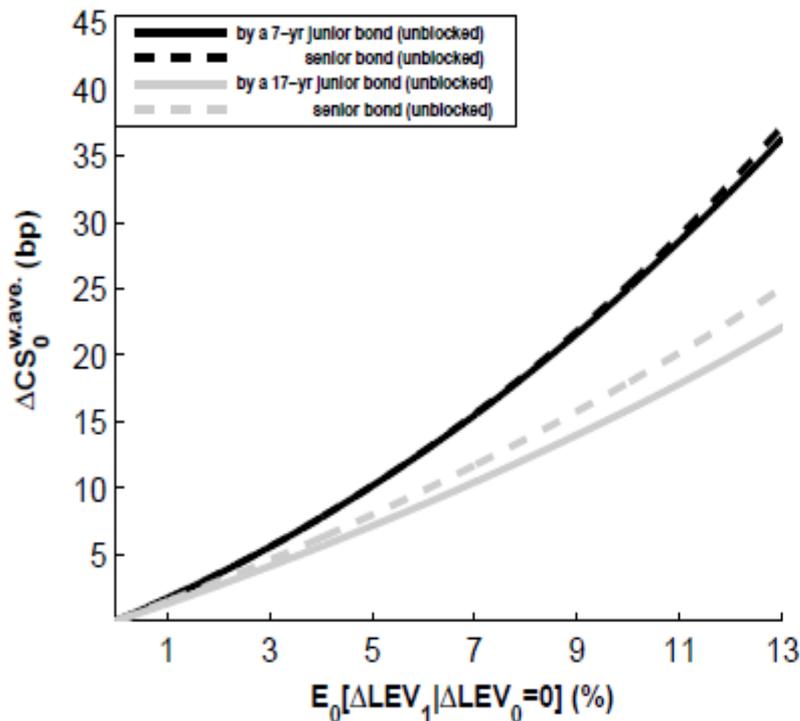
- **Debt structure:** (1) 5-yr bank loan (BD), 10-yr senior debenture (SD), 20-yr junior debenture (JD), with equal face value
(2) $BD \succ SD \succ JD$

Scenario: The firm now announces a new bond issuance, a new SD or JD

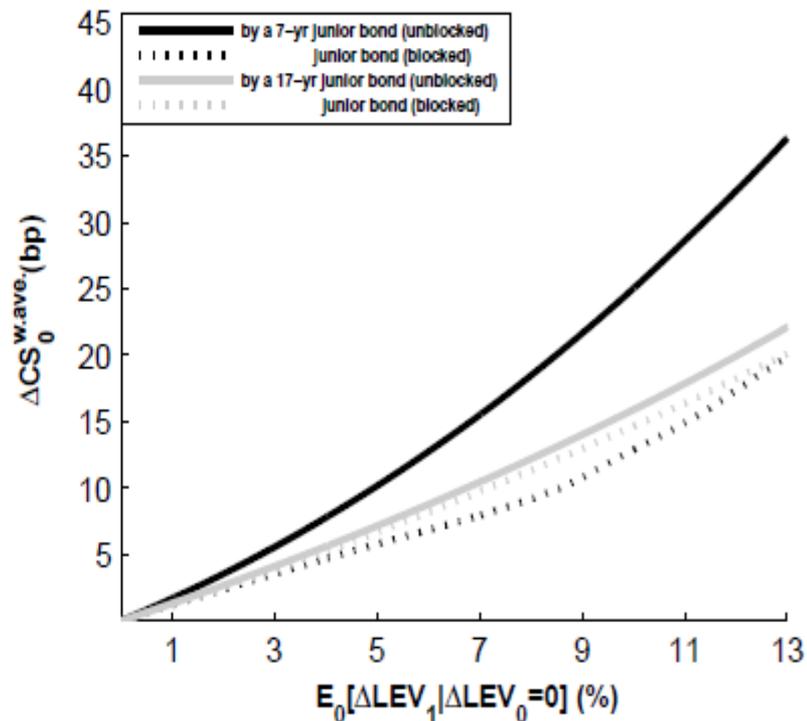


- **Debt structure:** (1) 5-yr bank loan (BD), 10-yr senior debenture (SD), 20-yr junior debenture (JD), with equal face value
(2) $BD > SD > JD$

Scenario: Investors expect that the firm will announce a new bond issuance, a new SD or JD, in a year



(the payments to new JD are blocked by BD and SD)



Appendix 2

the effect of rollover risk on a firm's long term bond

$\Delta S_{longterm} \uparrow$, the more the firm with larger proportion of
bonds to be rolled over

(focus on bond market illiquidity)

(Gopalan et al., 2014, Nagler, 2014)

the effect of rollover risk on a firm's long term bond

- Debt structure: (1) 1-yr BD, **16-yr SD**, 25-yr JD, given SD's face value
- (2) $BD > SD > JD$

Scenario: The firm will rollover the 1-yr BD

