Evaluating Corporate Bonds with Complex Debt Structure

Liang-Chih Liu
Department of Information Management and Finance
National Chiao Tung University

Joint work with Prof. Tian-Shyr Dai and Prof. Chuan-Ju Wang

December 4, 2014
1st Conference on Recent Developments in Financial Econometrics and Applications

Outline

- Motivation
- Main Contributions
- Assumptions and Numerical Implementation
  - **- model assumptions**
  - **- numerical implementation: trees and forests**
- Numerical Results and Empirical Implications
  - **- 4 scenarios and the corresponding empirical implications**
- Conclusions

Motivation

Motivation

- **A corporate bond is a popular financing or investment tool.**
  - outstandings in the US market: 2127 billions in 1996
  9617 billions in 2013 **(from SIFMA)**

- **Among valuation methodologies,**
  - the structural model of credit risk is a popular approach that associates *observed bond prices* with *firm-specific capital structure*
  (e.g., Merton(1974); Leland(1994), etc)

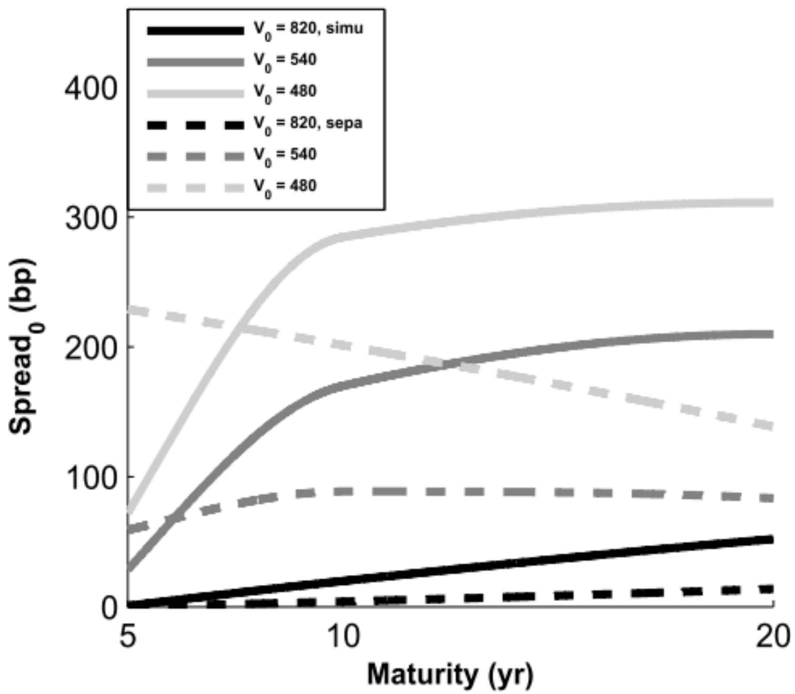
- **Regarding the capital structure,**
  - theoretical researches: equity + "oversimplified" debt
  e.g. equity + uniform debt (i.e., the debt containing only one bond)
  (e.g., Merton(1974); Black and Cox (1976); Leland(1994))

  - empirical observations: equity + the debt containing heterogeneous bonds
  (e.g., Rauh and Sufi (2010); Colla et al. (2013))

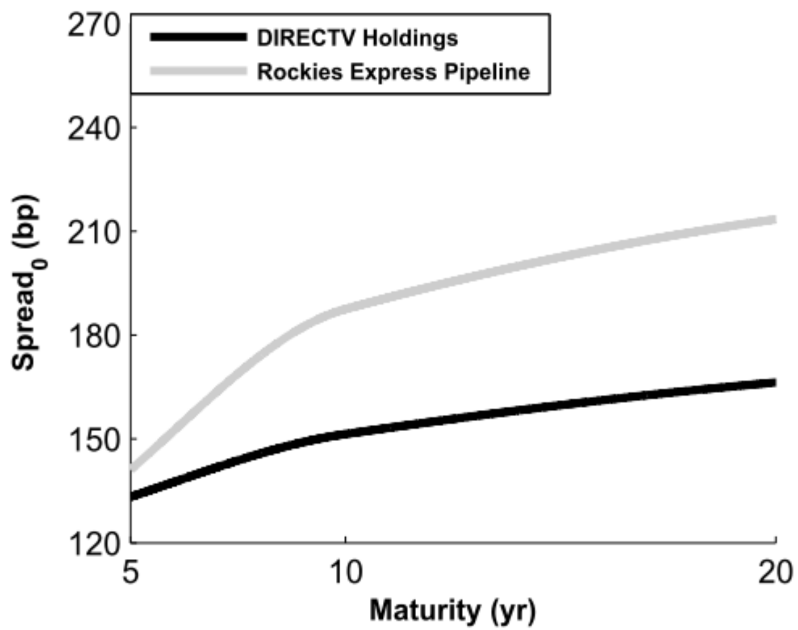
- **Proxy real world observations with structure models**
  - **for example, structural models with uniform debt separately evaluate each bond of the same firm that actually exists simultaneously**

    (e.g. Merton (1974), Kim et al. (1993), Briys and De Varenne (1997))

solid curve: bonds are simultaneously evaluated
dash curve: bonds are separately evaluated



real world observations



(Helwege and Turner, 1999 ; Huang and Huang, 2008)

- **To promote the empirical validity of structural models**

  - additional firm- or market-level factors are needed!

  (Jones et al., 1984; Collin-Dufresne et al., 2001)

  - the interdependence of bond issuers' financing and investment decisions is highlighted!

  (Barclay et al., 2003; Hackbarth and Mauer, 2012; Kuehn and Schmid, 2014)

- **Given *parameters* and *observed decisions*, we recognize the *observed debt structure* and pave another way to address that**

  - rather than being evaluated separately, corporate bonds issued by the same firm must be priced considering the presence of other simultaneously outstanding bonds of that firm. **(succeed Wang, Dai and Lyuu (2014))**

  (respond to Colla et al. (2013))

  - this may greatly reconcile structural models with empirical observations.

  (respond to Eom et al. (2004); Davydenko(2012))

  - our valuation framework will provide theoretical insights and concrete quantitative measurements on these empirical phenomena.

Main Contributions

- **A more intuitive valuation framework can be implemented!**

  - That is, corporate bonds of the same firm are priced considering the presence of other simultaneously existing bonds with different properties, such as different maturities, seniorities,..etc.

- **A structural model that characterizes the multidimensionality of corporate debt structure is developed!**

  - Four observable dimensions:

  (1) leverage ratio (2) maturity structure

  (3) priority structure (4) covenant structure

(Kisgen, 2006; Billett et al. 2007; Mauer et al., 2012; Gopalan et al., 2014)

- **To implement the structural model with debt structure that has observable time-dependent debt service payments,**

we propose a novel quantitative framework:
a forest with multi-layer trees

- **A bond is a security with time-dependent debt service payments**

- **To capture the contingent changes of the debt structure due to early redemption, we need more than one tree and make them work collaboratively as a forest**

- **It may be an alternative way to solve the unsolved problem in Jones et al. (1983) (i.e., the problem to evaluate *equity* and *the corresponding callable bonds* of the same firm)**

- **Numerous phenomena based on previous empirical researches are revisited through our valuation framework:**

[1] the impact of rollover risk through the tunnel of bond market illiquidity on the existing bonds
(He and Xiong, 2012; Gopalan et al., 2014; Nagler, 2014)

[2] the impact of a junior bond issuance to replace the existing bank loan on the existing senior bonds
(Ingersoll,1987; Linn and Stock, 2005)

[3] optimal call policy for callable bonds and call delay phenomenon
(Ingersoll 1977; Longstaff and Tuckman, 1994; King and Mauer, 2000; Acharya and Carpenter, 2002; Jacoby et al., 2010)

[4] the effect of including poison put covenants in target firms' bonds on bidders' costs of debt financing
(Cook and Easterwood, 1994; Cremers et al., 2007)

Assumptions and Numerical Implementation

- **Model Assumptions**
- **Numerical Implementation: trees and forests**

Model Assumptions

(A.1) There are no dividend payments or other disbursements to equity holders.

Ingersoll (1977); Collin-Dufresne et al.(2001); Avramov et al. (2007)

(A.2) Let a firm's asset volatility as the proxy of the firm's business risk and treat it as the result of the firm's contemporaneous investment and financing decisions subject to the covenant restrictions included in the previously issued bonds

Merton (1974)

(A.3) A firm is assumed to issue more than one bond at different times *to comprise its debt structure observed*; we treat the debt structure as the result of the firm's previous and current financing decisions.

(A.4) A firm files for bankruptcy and is liquidated immediately once it defaults on its debt obligation.

Ou et al.(2006) – Moody's definition of default
Bris et al.(2010) – Immediate liquidation

The remaining assets are then distributed according to absolute priority rules.

Bris et al.(2006) – No violation of absolute priority rule

- Denote the market value of the firm's asset at time t as V_t

- Under the structural model of credit risk, corporate bonds and the corresponding equity are contingent claims on the issuing firm's asset value, denoted as $B_1(t, V_t), \ldots, B_N(t, V_t)$, $E(t, V_t)$

- Under the risk-neutral valuation associated with the analysis of taking both the investment and financing decisions as given, the firm's asset value follows the dynamics: **Harrison and Kreps (1979)**

$$dV_t = (rV_t - P)\,dt + \sigma V_t\, dz, \qquad (1)$$

where

Attaoui and Poncet (2013)

[1] r is the long-term average risk-free rate;
[2] σ is the volatility of the firm asset value, representing the firm's business risk;
[3] dz is a standard Brownian motion;
[4] P is the firm's annual required payments made continuously

Merton (1974)

- We do not consider the shock of the firm's internal liquidity.

He and Xiong (2012)

- Rather than the minus term $-P$ in Eq. (1), the ***observed time-dependent payouts*** implied by the ***observed debt structure*** are needed to be captured faithfully.

- For example, let $C_{t_j}^{O}$ and $C_{t_{j+1}}^{O}$ be two required payouts made discr at
time t_j and t_{j+1}, $t_j < t_{j+1}$; t^- and t^+ denote the time immediate befo
and after a required payout.

Then $$\begin{cases} V_{t_j}^{+} = V_{t_j}^{-} - C_{t_j}^{O} \\ V_{t_{j+1}}^{+} = V_{t_{j+1}}^{-} - C_{t_{j+1}}^{O} \end{cases}, \qquad (2)$$

where $V_{t_{j+1}}^{-}$ is derived from $V_{t_j}^{+}$ and Eq. (1) given $P=0$.

- The firm determines to file for bankruptcy at time t^- once $V_{t}^{+} \leq 0$

- Previous literatures impose additional assumptions on P in Eq.(1) and default triggers

(a) There is a prespecified threshold value Π, $\Pi>0$.
The firm decides to file for bankruptcy at time t^- once $V_{t^+} \leq \Pi$

Black and Cox (1976), Longstaff and Schwartz (1995)

(b) **Geske (1977)** assumes $P=0$ for all $t>0$ if a firm is solvent.
The firm decides to file for bankruptcy at time t^- once $E(V_{t^+}, t^+) \leq 0$

(c) **Leland and Toft (1996)** assume $P=\delta V_t \, dt$ for all $t>0$ if a firm is solvent and propose stationary debt structure:

- The firm will rollover all its bonds to keep the number of its outstanding bonds, sums of bond principals and annual coupon payments unchanged.

- The equity holders will absorb all deficiencies in required payments, including the rollover losses, to prevent bankruptcy.

The firm decides to file for bankruptcy at time t^- once $E(V_{t^+}, t^+) \leq 0$

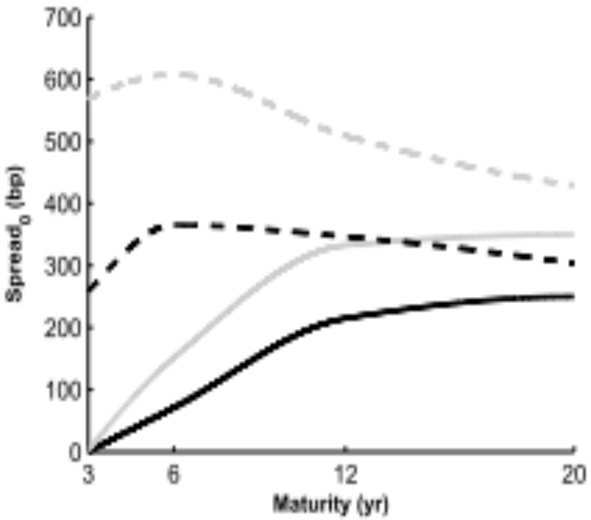
- However, these assumptions will lead to severe cross-sectional inaccuracy

 such as <u>hump-shaped</u> or <u>downward-sloping</u> credit spread curves.

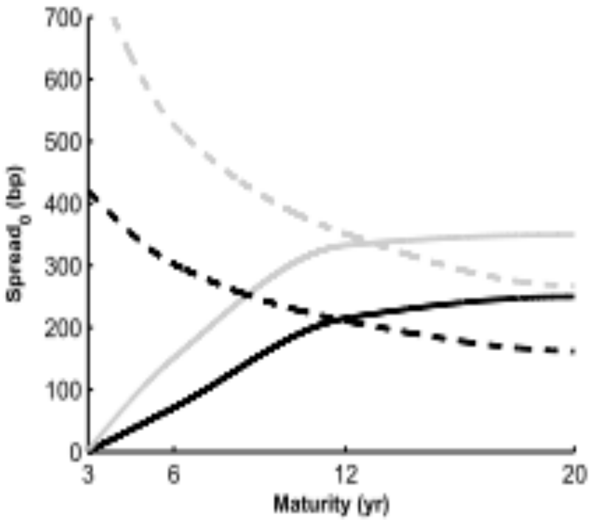
 (Lando, 2004; Eom et al, 2004; Davydenko, 2012)

For example, ***with the same parameters***,

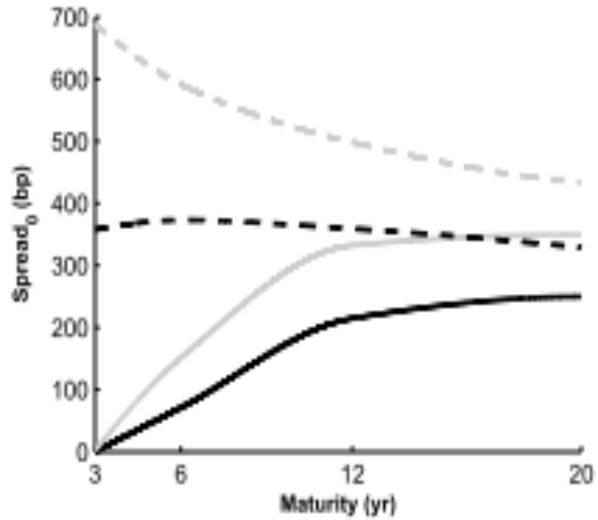
Solid curves: our models
Dashed curves: others



(a) VS. Prespesify a default boundary



(b) VS. Geske's framework



(c) VS. Leland and Toft's framework

(the detailed discussions are listed in our

Numerical Implementation
- binomial/trinomial trees

- CRR Trees for the lognormal diffusion process:

$$dV_t = rV_t\,dt + \sigma V_t\,dz$$

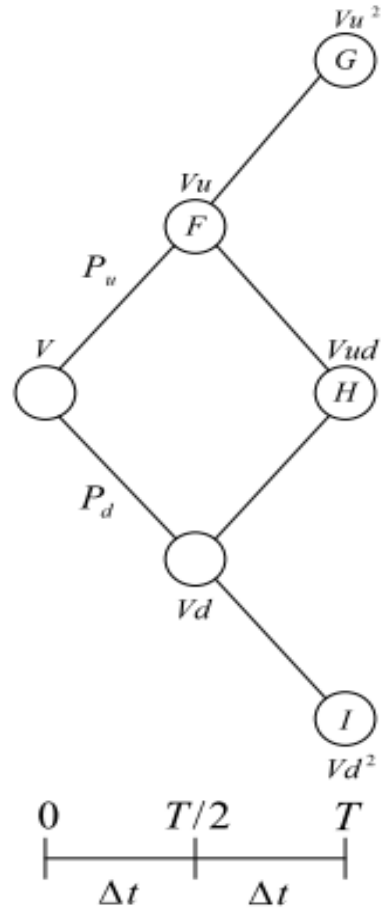
[1] Size of one time step: $\Delta t = T/n$

[2] 4 parameters: u, d, P_u , P_d :

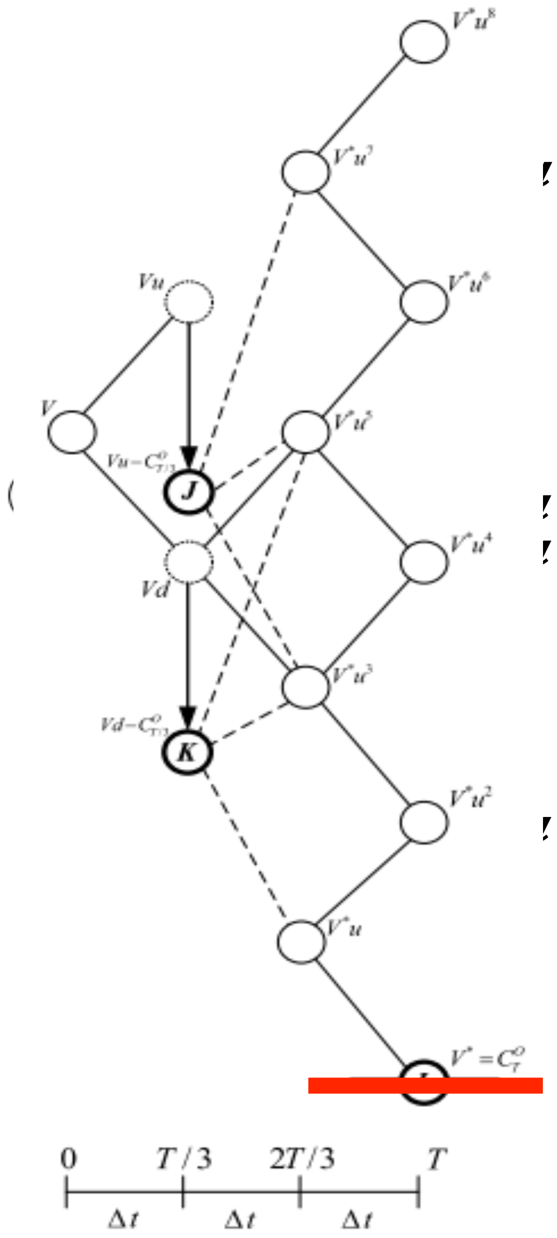
- $u = e^{\sigma\sqrt{\Delta t}}$, $d = 1/u$;

Cox, Ross and Rubinstein (1979)

- $P_u = e^{r\Delta t} - d/$



- The trinomial structure is used to deal with jumps in a firm's asset value and to coincide critical locations, such as default boundaries:



$+2\boldsymbol{\sigma}\sqrt{\Delta}\boldsymbol{t}$

$-2\boldsymbol{\sigma}\sqrt{\Delta}\boldsymbol{t}$

Default Boundary

- With feasible branching probabilities p_u , p_m , p_d that satisfying

$$\{ p_u\alpha + p_m\beta + p_d\gamma = 0 \quad p_u(\alpha)^2 + p_d(\gamma)^2 \quad p_u + p_m + p_d = 1 \quad = \sigma^2 \Delta$$

where

$\alpha \equiv \beta + 2\sigma\sqrt{\Delta}t$

$\beta \equiv \mu - \mu$

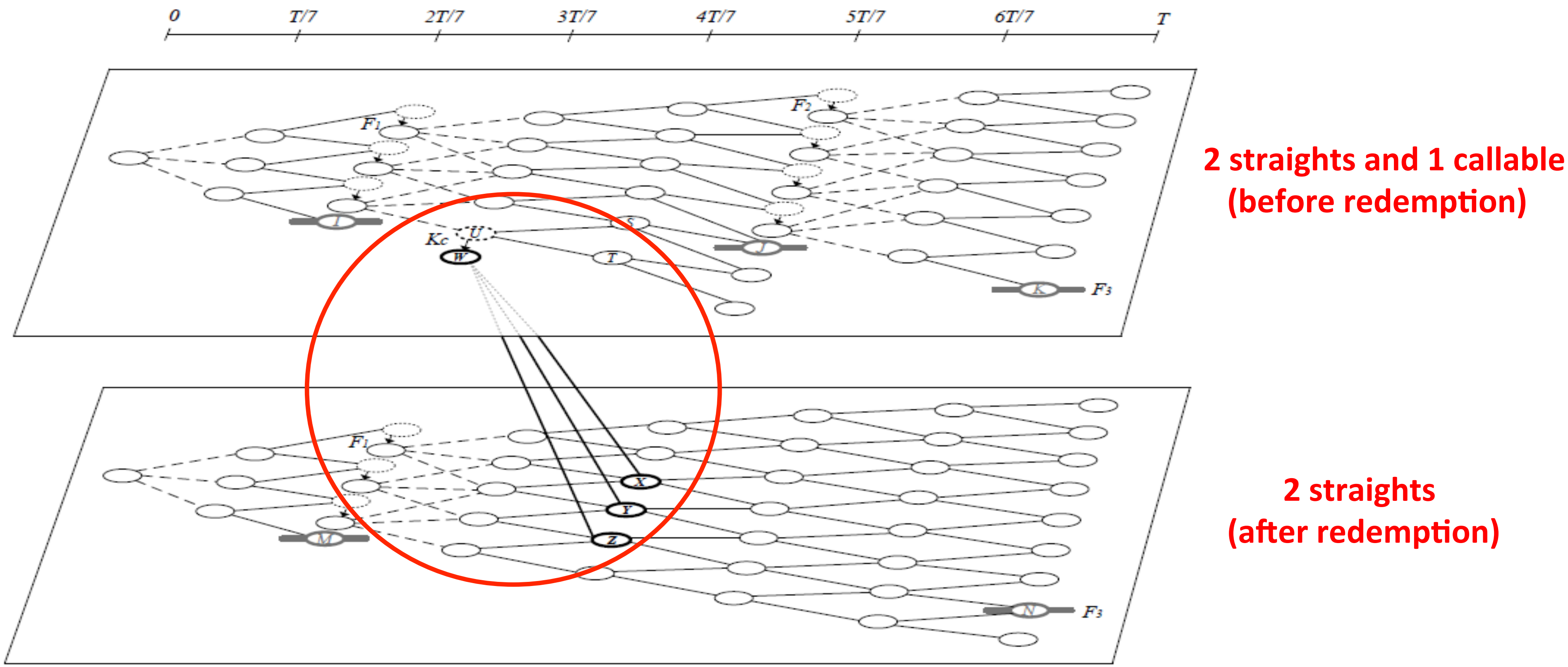
$\gamma \equiv \beta - 2\sigma\sqrt{\Delta}t$

Dai and Lyuu (2010)

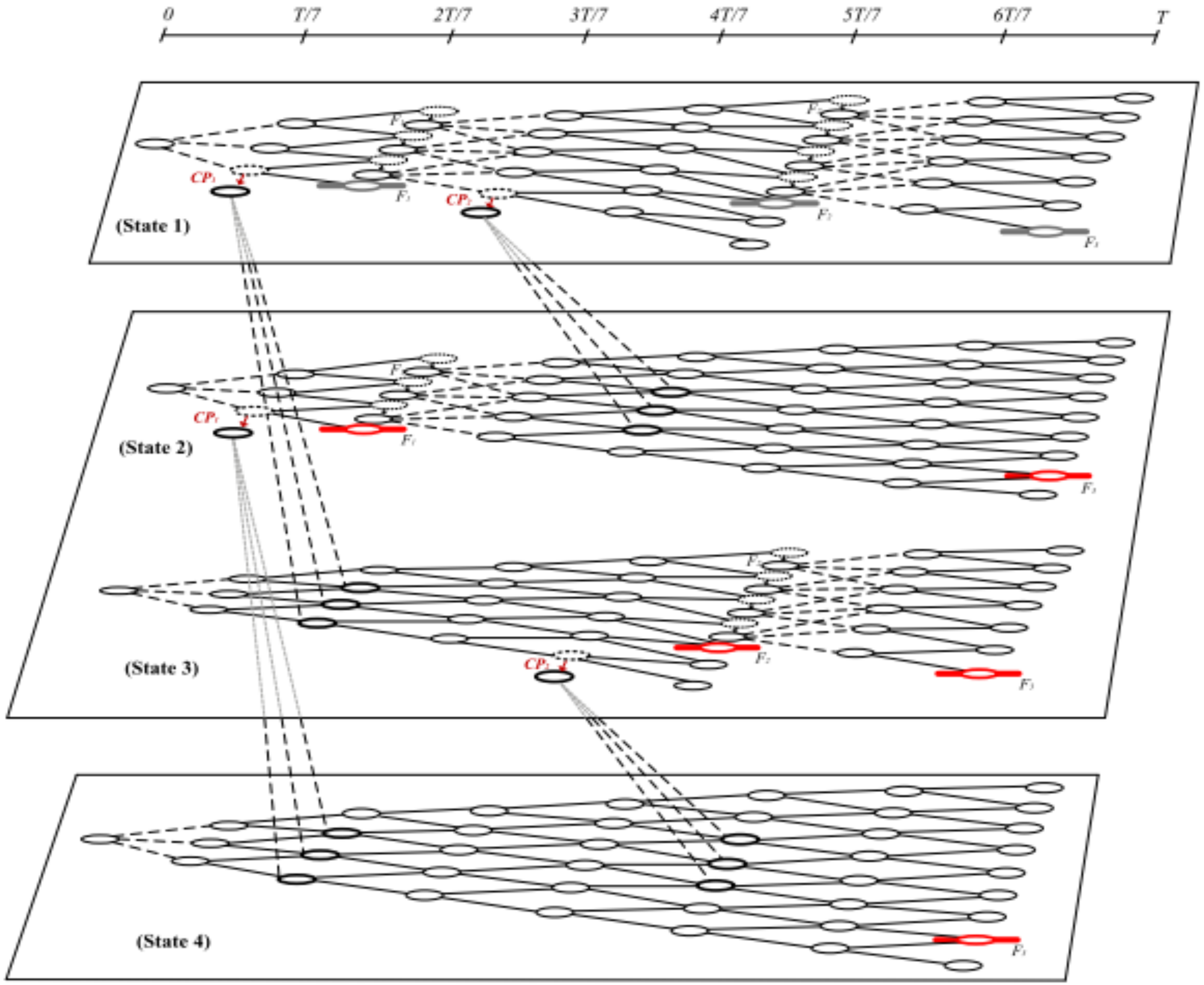
Numerical Implementation
- a forest

- To capture the contingent changes of the debt structure due to early redemption, we need a forest.

- For example, a firm issues 2 straight bonds and 1 callable bond,



A forest with two-layer trees



A forest with three-layer trees

Numerical Results and Empirical Implications

Robustness:

The following numerical results are generated by the aforementioned numerical models confirmed to

(1) converge smoothly as the number of the time steps in a certain period increases Duffie (1996)

(2) produce empirically observed spread-rate, spread-volatility and spread-leverage relations

Duffee (1998), Collin-Dufresne et al.(2001), Avramov et al. (2007), Flannery et al.(2012)

(all the details about the robustness check are listed in our paper)

4 scenarios visited:

[1] the effect of rollover risk on existing bonds
[2] the effect of junior bond issuances to replace bank loans on senior bonds
[3] the optimal call policy and call delay phenomena
[4] the effect of including poison put covenants on bidders' cost of debt financing

the effect of rollover risk on a firm's existing bonds through the tunnel of bond market illiquidity

Two key parameters incorporated:

(1) bond transaction cost k (2) market condition proxy ξ

(He and Xiong, 2012)

Valuation effect:

(1) the spreads of not only the existing long-term bonds (with remaining time to maturity more than 16 years) but whole term structure increases with the amounts of bonds that will be rolled over (bonds maturing within 1 year)

(2) such effect would be more pronounced

[a] when the firm is unhealthy,

[b] when the maturing bond is replaced by a short-term bond,

[c] when the capital market is in recession

(Gopalan et al., 2014; Nagler, 2014)

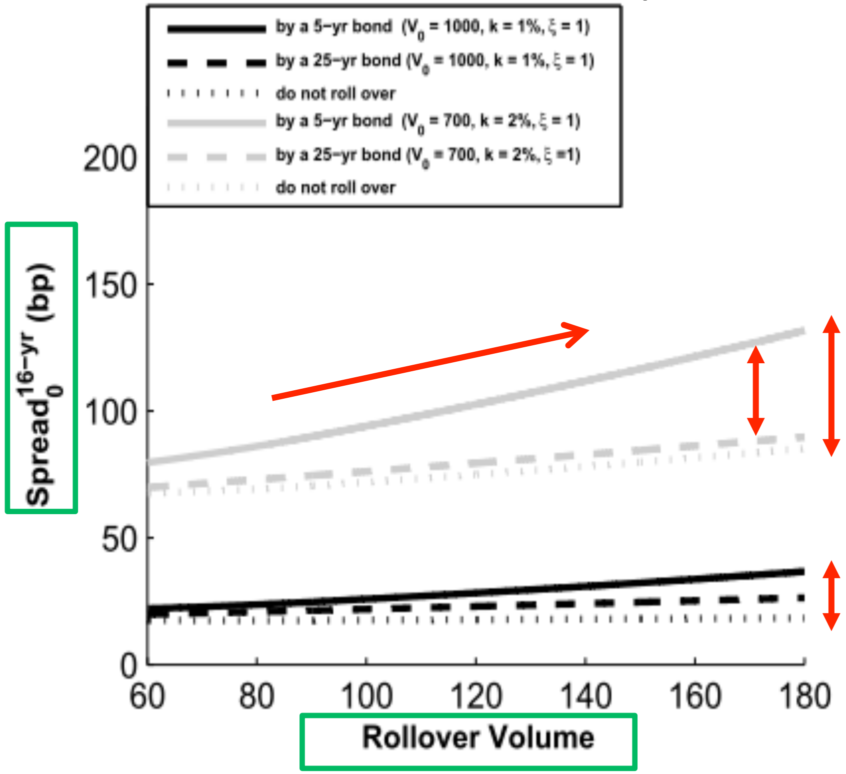
- **Debt structure:** (1) 1-yr B_1 , **10-yr** $\boldsymbol{B_2}$, **16-yr** $\boldsymbol{B_3}$, **20-yr** $\boldsymbol{B_4}$, 30-yr B_5

  (2) given the face of $\boldsymbol{B_2}$, $\boldsymbol{B_3}$, $\boldsymbol{B_4}$

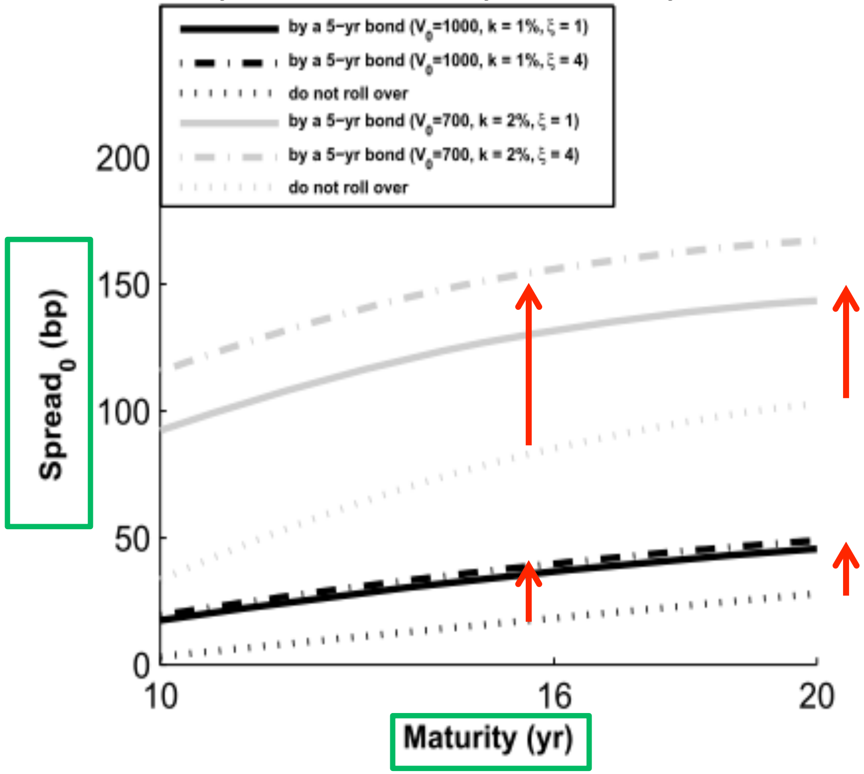
  (3) 5 equal priority bonds

Given total face value of the bonds

Scenario: A firm will replace the 1-yr B_1 by another 5-yr or 25-yr bond



the spreads of the given long-term bond



the spreads of the whole term structure

the effect of a junior bond issuance (NJB) to replace the existing bank loan (BB) on the existing senior bond (SB)

Valuation effect:

Given bank loans are the most senior bond type in the firm,

(Welch, 1997; Gorton, 2000)

(1) The spread of the existing senior bond decrease ($\Delta Spread_{0}^{SB} < 0$);

such effect would be more pronounced with the replacement size

(∵ the relative priority of the senior bond is improved)

(2) ceteris puribus, the maturity of the junior bond makes small difference "on average" until the firm becomes relatively unhealthy

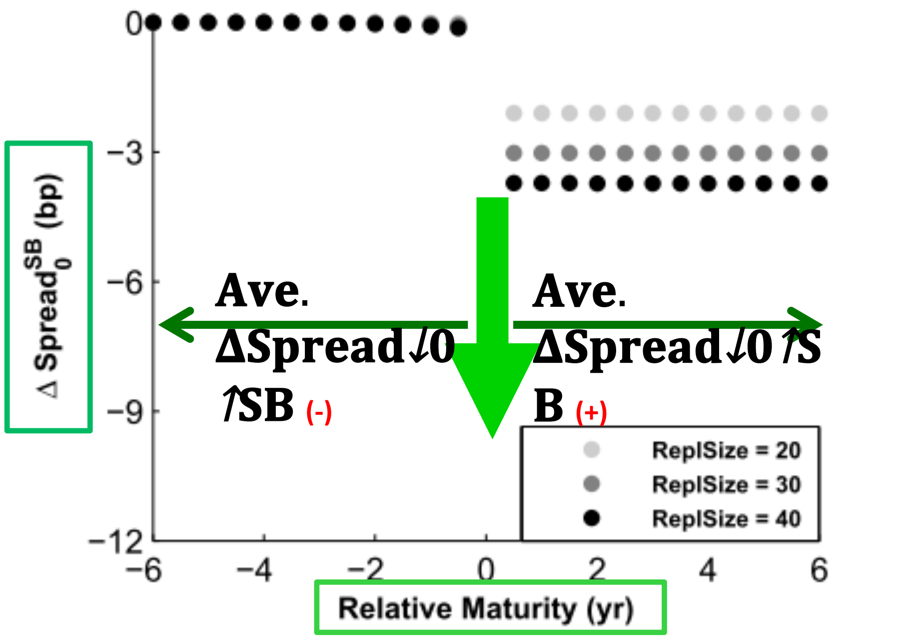
(∵ the existence of payment blockage covenant)

(Ingersoll, 1987)
(Linn and Stock, 2005)

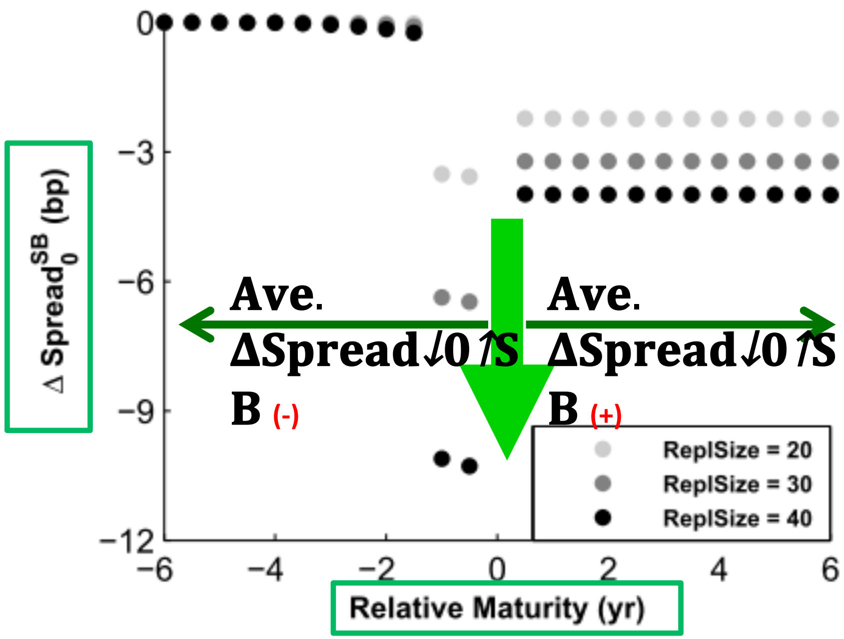
- **Debt structure:** (1) BB, **10-yr SB**, 20-yr JB
  (2) given the face value of the SB
  (3) BB ≻ SB ≻ JB

Scenario: A firm now issues an otherwise identical NJB to replace the existing BB

Given total face value of bonds and the firm's credit quality

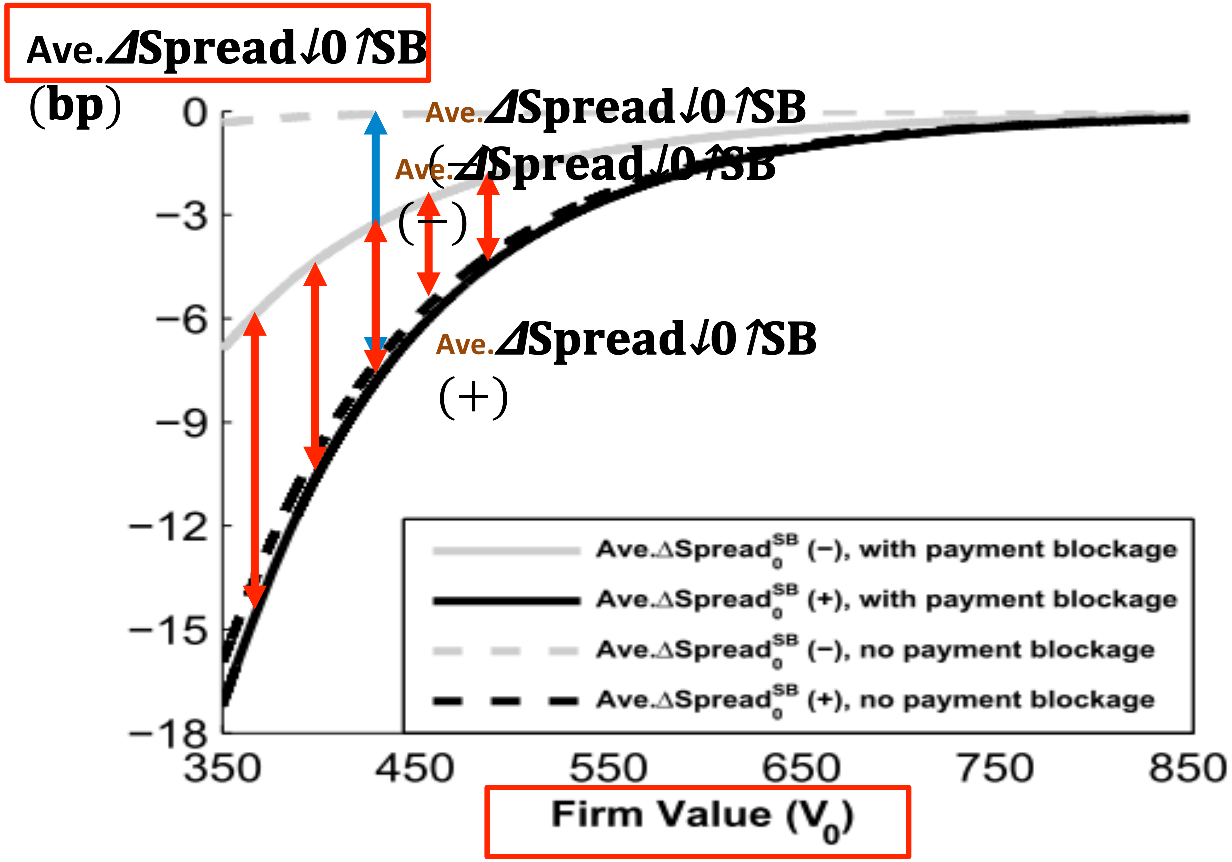


SB WITHOUT the payment blockage covenant



SB WITH the 1-yr payment blockage covenant

Given total face value of bonds and the replacement size



unhealthier firm

healthier firm

Solid Curves
(with payment blockage)

Dashed Curves
(no payment blockage)

the maturity of the new junior bond makes small difference "on average"
<u>until the firm becomes relatively unhealthy</u>

the optimal call policy and call delay phenomenon

the optimal call policy:

redeeming bonds to maximize equity holders' value

Given other thing being equal,

(a) low level of interest rates precipitate calls

(b) long (high-coupon) callable bonds are called prior to short (low-coupon) callable bonds (King and Mauer, 2000)

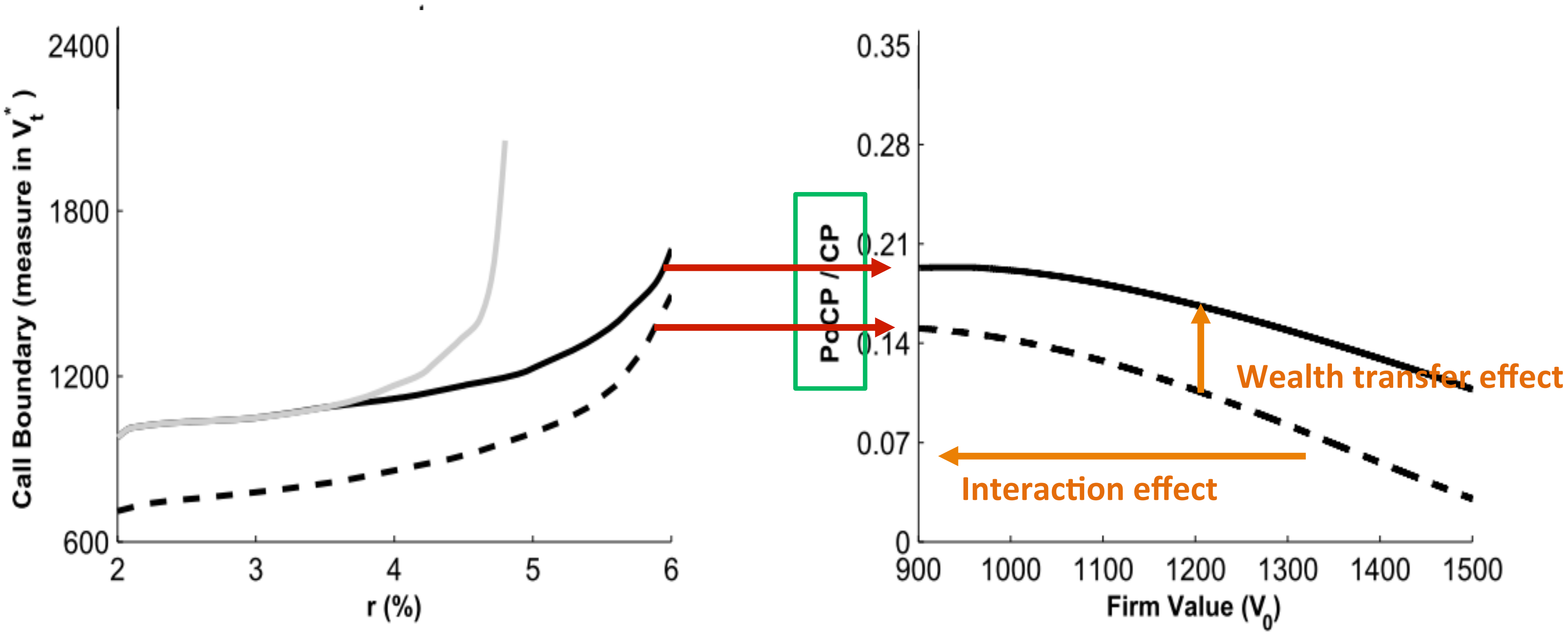
(c) the deterioration in the issuing firm's creditworthiness delays calls. (Jacoby et al., 2010)

the reason may be

- (c.1) interaction effect (single-bond case) (Acharya and Carpenter, 2002)
- (c.2) wealth transfer effect (multiple-bond case) (Longstaff and Tuckman, 1994)

- **Debt structure:** single-bond case: a 16-yr callable
  multiple-bond case: 5-yr, 10-yr, 16-yr, 20-yr and 30-yr bonds,
  the 10-yr is a short callable.
  the 16-yr is a long callable.

the single-bond case vs. the multiple-bond case



low level of interest rate

given the level of interest rate

the effect of including poison put covenants in target firms' bonds on bidders' cost of debt financing when finishing LBOs

Valuation effect:

By changing the bidder's debt maturity structure,
poison put covenants significantly increase the bond value of the target firm by increasing the bidder's costs of debt financing.

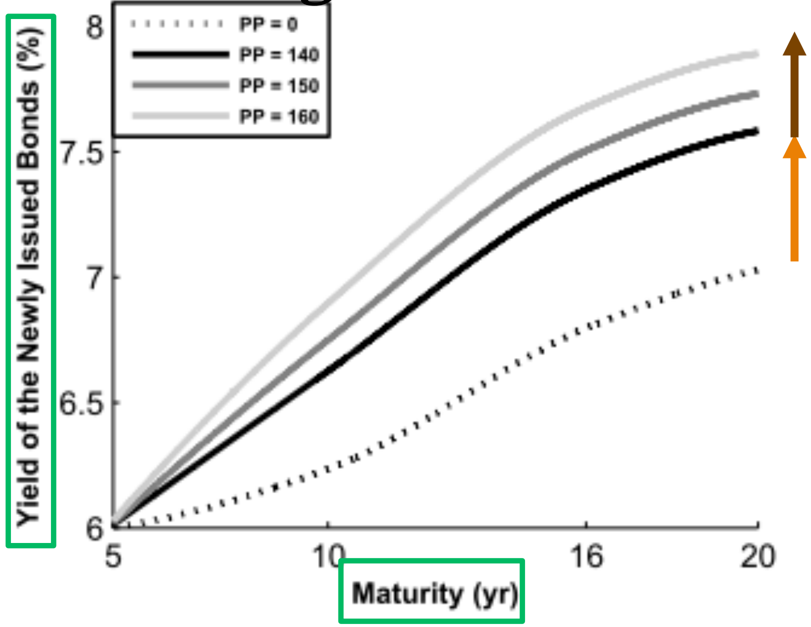
(Cremers et al., 2007)

- **Scenario:** A bidder finishes a LBO by issuing 4 otherwise identical bonds

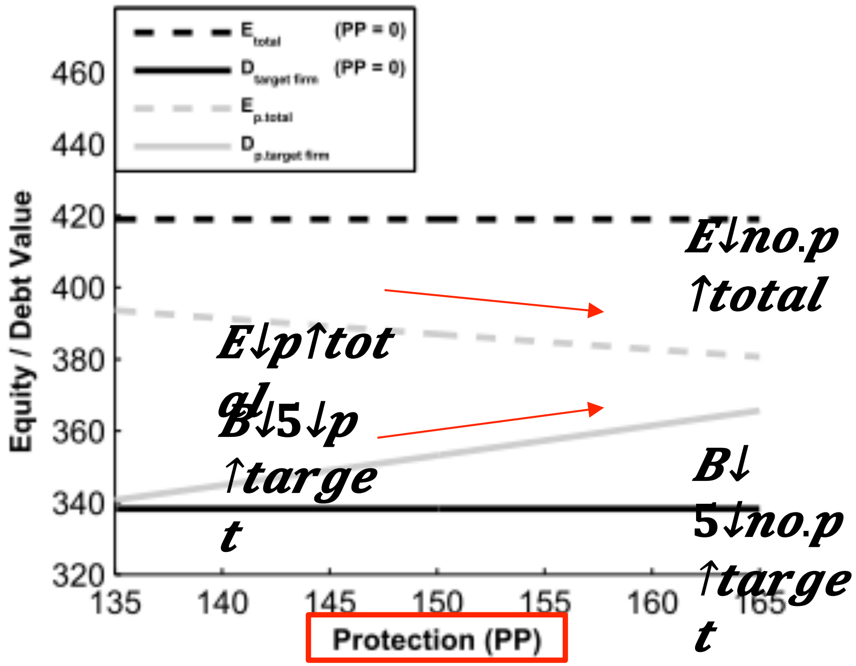
5-yr B_1, 10-yr B_2, 16-yr B_3 and 20-yr B_4

Debt structure: (1) 5-yr B_1, 10-yr B_2, 16-yr B_3, 20-yr B_4, 30-yr B_5^{bidder}, 30-yr B_5^{target}

(2) B_1 ,..., B_4 and B_5^{bidder} are senior to B_5^{target}



the bidder's costs of debt financing



Conclusions

- A structural model of credit risk with debt structure simultaneously considering four observable dimensions is developed:

  - **[1]** leverage ratio **[2]** maturity structure
  [3] priority structure **[4]** covenant structure

  - the **forest** can capture the contingent changes of debt structure, such as the early redemption of bonds with call provisions or poison put covenants

- Compared with existing structural models, our valuation framework can produce the results that are more consistent with the observations documented already in empirical studies.

- including 8 observations:

[1] upward sloping credit spread curves

[2] spread-rate relation

[3] spread-volatility relation

[4] spread-leverage relation

[5] the effect of rollover risk on existing bonds

[6] the effect of junior bond issuances to replace bank loans on senior unsecured bonds with or without payment blockage covenants

[7] call delay phenomena

[8] the effect of including poison put covenants on bidders' cost of debt